# Four Roots

**Submission deadline: April $30^{\text{th}}$ 2018**

Determine $m$ so that the equation

$$x^4 - (3m+2)x^2 + m^2 = 0$$

has 4 real roots in arithmetic progression.

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**Solution**

Clearly $x^2 = ((3m+2) \pm \sqrt{5m^2+12m+4})/2$. Let $A = (3m+2)/2$ and $B = \sqrt{5m^2+12m+4}/2$. Then the roots of the polynomial arranged as an increasing sequence is

$$-\sqrt{A+B}, -\sqrt{A-B}, \sqrt{A-B}, \sqrt{A+B}.$$

Since

$$\sqrt{A+B} - \sqrt{A-B} = \sqrt{A-B} - (-\sqrt{A-B})$$

we get

$$9(A-B) = A+B.$$

Which yields

$$4(3m+2) = 5\sqrt{5m^2+12m+4}$$

Squaring the equation above we get

$$19m^2 - 108m - 36 = 0,$$

which has the roots 6 and $-6/19$.

The root $m = 6$ results in the arithmetic sequence

$$-3\sqrt{2}, -\sqrt{2}, \sqrt{2}, 3\sqrt{2}$$

and $m = -6/19$ yields

$$-3\sqrt{\frac{2}{19}}, -\sqrt{\frac{2}{19}}, \sqrt{\frac{2}{19}}, 3\sqrt{\frac{2}{19}}$$