A Squared Divisor

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If n is an integer greater than 1, then show that $n^{n-1} - 1$ is divisible by $(n-1)^2$.

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NOTE. One of the solutions we received was valid only for odd integers.

Discussion:

The $n = 2$ case is trivial. Assume that $n > 2$.

$$\begin{aligned} n^{n-1} - 1 &= (n-1)(n^{n-2} + n^{n-3} + \cdots + n + 1) \\ &= (n-1)(n^{n-2} - 1 + n^{n-3} - 1 + \cdots + n - 1 + n - 1) \end{aligned} \tag{1}$$

Since each $n^k - 1$ where $k > 1$, can be factored as $(n-1)(n^{k-1} + \cdots + 1)$, it follows that $(n^{n-2} - 1 + n^{n-3} - 1 + \cdots + n - 1 + n - 1) = (n-1)M$ for some integer M. Thus $n^{n-1} - 1 = (n-1)^2 M$.