# Primes.

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Find all positive integers $a, b$ such that $a^4 + 4b^4$, is prime.

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Discussion
Clearly $a^4 + 4b^4 = (a^2 + 2b^2)^2 - 4a^2b^2$. Thus,

$$a^4 + 4b^4 = ((a^2 + 2b^2) - 2ab)((a^2 + 2b^2) + 2ab)$$

For any two real numbers $a$ and $b$, we have that $(a^2 + b^2) - 2ab \geq 0$, and equality is attained only if $a = b$. Thus, for distinct natural numbers $a$ and $b$, it is clear that $(a^2 + b^2) - 2ab \geq 1$, and hence, $(a^2 + 2b^2) - 2ab \geq 2$. If $a = b$, then $a^4 + 4b^4 = 5a^4$, therefore $a = 1$ yields the only prime number. For any natural numbers $a$ and $b$ it is clear that $(a^2 + 2b^2) + 2ab \geq 5$. Thus, $a^4 + 4b^4$ is prime only when $a = 1$ and $b = 1$.