Divisible Difference

Submission deadline: March 27th 2023

Let n be a natural number greater than 1. Further let $a_1, \cdots, a_n$ be a sequence of any n distinct natural numbers. Prove that in the sequence above, either there is a number that is divisible by n, or there are two distinct numbers whose difference is divisible by n.

The problem was solved by

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Discussion:

Clearly

$$a_i = nk_i + r_i,\ 0 \leq r_i \leq n-1$$

for $i = 1, \cdots, n$.

If $r_j = 0$, for some j, then a_j is divisible by n.

If $r_i \neq 0$, for all i, then there are not more than $n-1$ distinct values for all the remainder terms. However, there are n numbers in the sequence, hence there are at least two terms a_m and a_l with $r_m = r_l$. It easily follows that $a_m - a_l$ is divisible by n.