Constant Product

Submission deadline: April 30^{th} 2019

Let f be a positive valued continuous function defined on the interval $[0, 2019]$ such that $f(x) \cdot f(2019 - x) = 1$ for all x in its domain. Evaluate

$$\int_0^{2019} \frac{1}{1 + f(x)} dx$$

The problem was solved by

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Discussion

Let

$$I = \int_0^{2019} \frac{1}{1+f(x)} dx$$

and it easily follows that

$$I = \int_0^{2019} \frac{f(2019-x)}{1+f(2019-x)} dx$$

Change of variables with $t = 2019 - x$, yields

$$I = \int_0^{2019} \frac{f(t)}{1+f(t)} dt.$$

Thus

$$I = \int_0^{2019} dt - \int_0^{2019} \frac{1}{1+f(t)} dt$$

Therefore $2I = 2019$.