AUS | الجامعة الأميركية في الشارقة
American University of Sharjah

The Fourth International Conference on Mathematics and Statistics | ICMS'25

February 20 – 22, 2025

Book of Abstracts

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Welcome to the AUS-ICMS25

On behalf of the Department of Mathematics and Statistics at the American University of Sharjah, it is my pleasure to present this Book of Abstracts for the Fourth International Conference on Mathematics and Statistics (AUS-ICMS25). Building on the success of our previous conferences in 2010, 2015, and 2020, this year's event brings together mathematicians and statisticians from around the world to share and discuss innovative research in diverse fields of mathematics and its applications.

We are honored to host over 200 participants from more than 40 countries, all contributing to a rich and dynamic exchange of ideas. This year's program features three distinguished keynote lectures by internationally renowned mathematicians, along with special and general sessions on contemporary, emerging, and fundamental topics—including Algebraic Topology, Statistical Learning, Financial Mathematics, Mathematical Modeling in Biology, Algebra, Analysis, Applied Mathematics, Graph Theory, and Computational Mathematics, among others.

This Book of Abstracts features numerous submissions reflecting the diversity of topics at AUS-ICMS25, with papers spanning both theoretical and applied aspects of mathematics and statistics. We hope these presentations will provide valuable insights and inspire further discussions throughout the conference.

I would like to express my sincere gratitude to the American University of Sharjah for its unwavering support. This conference is organized in collaboration with the Society for Industrial and Applied Mathematics (SIAM) and the Institute of Mathematical Statistics (IMS). I would also like to thank McGraw Hill, Cengage, and Wiley for their generous sponsorship, as well as our keynote speakers, special session organizers, and all presenters for their invaluable contributions. A special thanks to our dedicated organizing team and student volunteers, whose efforts have made this conference possible.

I wish to thank all participants for being part of AUS-ICMS25. Your presence and engagement make this conference a platform for inspiring discussions and meaningful collaborations. I hope you find the conference both intellectually stimulating, professionally rewarding, and a chance to forge new collaborations.

Abdul Salam Jarrah
Chair, AUS-ICMS25
Head, Department of Mathematics and Statistics
American University of Sharjah

ICMS 25 ORGANIZING COMMITTEE

General Chair: Abdul Salam Jarrah, SIAM Representative

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- Ayman Alzaatreh
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Sponsorship: Saadia Khouyibaba

Daily Schedule for AUS-ICMS25

Time	Day 1, February 20
08:00 – 09:30	Registration
09:30 – 10:00	Opening Ceremony
10:00 – 11:00	Plenary I
11:00 – 11:30	Coffee Break
11:30 – 12:30	Plenary II
12:30 – 12:40	Photo Op
12:40 – 14:00	Lunch Break
14:00 – 16:00	Session A
16:00 – 16:30	Coffee Break
16:30 – 18:00	Session B

Time	Day 2, February 21
08:00 – 09:00	Registration
09:00 – 10:00	Plenary III
10:10 – 10:30	Coffee Break
10:30 – 12:30	Session C
12:30 – 14:00	Lunch Break
14:00 – 16:00	Session D
16:00 – 16:30	Coffee Break
16:30 – 18:00	Session E
19:00 – 22:00	Gala Dinner

Time	Day 3, February 22
08:00 – 09:00	Registration
09:00 – 10:00	Session F
10:00 – 10:30	Coffee Break
10:30 – 12:30	Session G
12:30 – 14:00	Lunch
14:00 – 16:00	Session H

SPECIAL SESSIONS

SS1 - Algebraic Topology and Applications

SS2 - Applied and Computational Analysis

SS3 - History of Mathematics

SS4 - Mathematics Education

SS5 - Modeling and Simulation of Biological Systems

SS6 - Operator Theory and Harmonic Analysis

SS7 - Rings, Monoids, Module Theory and Graphs from Rings

SS8 - Statistical Learning and Data Science

SS9 - Statistics and Data Science for Digital Finance on Tokenomics

GENERAL SESSIONS

GS1 – Algebra, Number Theory & Applications

GS2 - Analysis

GS3 - Applied Mathematics & Differential Equations

GS4 – Graph Theory, Coding Theory & Cryptography

GS5 - Computational Mathematics & Numerical Analysis

GS6 – Statistics & Machine Learning

DAY 1: Thursday, February 20, 2025

Time	DETAILED SCHEDULE										
8:00 - 9:30	**Registration** Main Building, Mezzanine, Dining Hall										
9:30 - 10:00	Opening Main Building, Hall A										
10:00 - 11:00	**Plenary I** **Keynote Speaker:** Dr. Kathryn Hess Bellwald **Title:** Topological Insights in Neuroscience Main Building, Hall A										
11:00 - 11:30	Coffee Break Main Building, Mezzanine, Dining Hall										
11:30 - 12:30	**Plenary II** **Keynote Speaker:** Dr. Martin Bohner **Title:** Generalized Periodicity Main Building, Hall A										
12:30 – 12:40	Photo Op Infront of Chemistry Building										
12:40 - 14:00	LUNCH Physics Gazebo										
14:00 - 16:00	**SESSION A**										
Rooms	**CHM113**	**CHM226**	**CHM227**	**NAB004**	**NAB005**	**NAB006**	**NAB007**	**NAB008**	**NAB009**	**NAB010**	
	SS6		SS1				SS7	SS4			
16:00 - 16:30	Coffee Break Chemistry Rotunda										
16:30 - 18:00	**SESSION B**										
Rooms	**CHM113**	**CHM225**	**CHM226**	**CHM227**	**NAB004**	**NAB005**	**NAB006**	**NAB007**	**NAB008**	**NAB009**	**NAB010**
	SS6	SS2	Cengage Presentation	SS1				SS7	SS4	GS6	GS2

DAY 2: Friday, February 21, 2025

Time	DETAILED SCHEDULE										
8:00 - 9:00	Registration Main Building, Mezzanine, Dining Hall										
9:00 - 10:00	**Plenary III** **Keynote Speaker:** Dr. Edriss Titi **Title:** Rigorous Analysis and Numerical Implementation of Nudging Data Assimilation Algorithms Main Building, Hall A										
10:00 - 10:30	Coffee Break Chemistry Rotunda										
10:30 - 12:30	**SESSION C**										
Rooms	**CHM113**	**CHM225**	**CHM226**	**CHM227**	**NAB004**	**NAB005**	**NAB006**	**NAB007**	**NAB008**	**NAB009**	**NAB010**
	SS6	SS2	SS8	SS1	GS4	SS5	SS9	SS7	SS3	GS6	GS3
12:30 - 14:00	LUNCH Physics Gazebo										
14:00 - 16:00	**SESSION D**										
Rooms	**CHM113**	**CHM225**	**CHM226**	**CHM227**	**NAB004**	**NAB005**	**NAB006**	**NAB007**	**NAB008**	**NAB009**	**NAB010**
	SS6	SS2	SS8	SS1	GS4	SS5	SS9	SS7	GS5	GS6	GS3
16:00 - 16:30	Coffee Break Chemistry Rotunda										
16:30 - 18:00	**SESSION E**										
Rooms	**CHM113**	**CHM225**	**CHM226**	**CHM227**	**NAB004**	**NAB005**	**NAB006**	**NAB007**	**NAB008**	**NAB009**	**NAB010**
		SS2	McGraw Presentation	SS1				GS1		GS6	GS3

DAY 3: Saturday, February 22, 2025

Time	DETAILED SCHEDULE										
8:00 - 9:00	Registration Chemistry Rotunda										
9:00 - 10:00	SESSION F										
Rooms	CHM113	CHM225	CHM226	CHM227	NAB004	NAB005	NAB006	NAB007	NAB008	NAB009	NAB010
		SS2		SS1							
10:00 - 10:30	Coffee Break Chemistry Rotunda										
10:30 - 12:30	SESSION G										
Rooms	CHM113	CHM225	CHM226	CHM227	NAB004	NAB005	NAB006	NAB007	NAB008	NAB009	NAB010
		SS2	Wiley Presentation	SS1							
12:30 - 14:00	LUNCH Physics Gazebo										
14:00 - 16:00	SESSION H										
Rooms	CHM113	CHM225	CHM226	CHM227	NAB004	NAB005	NAB006	NAB007	NAB008	NAB009	NAB010
		SS2									

DAY 1: February 20, 2025

Special Session A: Algebraic Topology (SS1) Chair: Sadok Kallel Thursday February 20 Room: CHM227			
Time	**ID**	**Title**	**Speaker**
14:00 – 14:50	1228	Many Partitions of Mass Assignments	Pavel Blagojevic
15:00 – 15:50	1250	Equivariant Bordism In Dimension 2 And the Evenness Conjecture of Equivariant Unitary Bordism	Andres Angel

Special Session A: Math Education (SS4) Thursday February 20 Chair: Jason Johnson Room: NAB008			
Time	**ID**	**Title**	**Speaker**
14:00 – 14:25	1033	The Impact of Interactive Digital Content Based on the Model in Developing Producing Educational Aids Skills	Feras Shatat
14:25 - 14:50	1195	Online Platforms for Conceptual Teaching? The Case of MyMathLab	Haitham Solh
14:50 - 15:15	1148	Uncertainties in Measuring the Course Learning Outcomes: Fuzzy Sets Approach	Jalalidin Jaenbai
15:15 - 15:40	1239	A Preliminary Study on Mathematics Through a Cultural Lens: Surveying the Role of Ethnomathematics in Teaching	Jason Johnson
15:40 - 16:05	1210	Advancing Mathematical Proficiency and Teacher Preparation: Insights from The Mental Starters Assessment Project	Lise Westaway

Special Session A: Operator Theory and Harmonic Analysis (SS6) Thursday February 20 Chair: Alexey Karapetyants Room: CHM113			
Time	**ID**	**Title**	**Speaker**
14:00-14:20	1197	Multilinear Sobolev Inequalities for Fractional Integrals Defined by Measure, and Related Topics	Alexander Meskhi
14:25-14:45	1087	Atomic Decomposition for the Bourgain-Brezis-Mironescu Space	Luigi D'onofrio
14:50-15:10	1090	Strong and Weak Associated Reflexivity of Certain Function Classes	Vladimir Stepanov
15:15-15:35	1251	Commutants of certain Classes of Toeplitz Operators	Issam Louhichi
15:40-16:00	1248	Hausdorff Operators on an Interval: Boundedness in Weighted Lebesgue Spaces	Alexey Karapetyants

Special Session A: Rings, Monoids, Module Theory, and Graphs from Rings (SS7) Thursday February 20 Chair: John Meakin Room: NAB007			
Time	**ID**	**Title**	**Speaker**
14:00 – 14:25	1131	Monoids Of Modules	Roger Wiegand
14:25 – 14:50	1145	Direct Projective Modules and Their Generalizations	Ashok Ji Gupta
14:50 – 15:15	1098	Study On Variants of Chain Condition in Modules and Rings	Manoj Kumar Patel
15:15 – 15:40	1093	Schreier And *-Modules	Ahmad Yousefian Darani
15:40 – 16:00	1146	Prime-Coefficient Graphs from The Gaussian Ring of Integers Modulo N	Mohamad Jawad Kaddoura

Sponsor Session B Chair: Saadia Khouyibaba Thursday February 20 Room: CHM226			
Time	**ID**	**Title**	**Speaker**
16:30 – 5:00	3002	Cengage Presentation	

Special Session B: Algebraic Topology (SS1) Chair: Sadok Kallel Thursday February 20 Room: CHM227			
Time	**ID**	**Title**	**Speaker**
16:30 – 17:20	1229	Pseudo-Anosov Maps and Integer Permutations	Ahmad Rafiqi
17:30 – 18:00	1222	Hochschild-Serre Spectral Sequence for Lie Conformal Algebras	Noufe Aloudah

Special Session B: Math Education (SS4) Thursday February 20 Chair: Jason Johnson Room: NAB008			
Time	**ID**	**Title**	**Speaker**
16:30 – 16:55	1241	Charting New Frontiers: Confronting the Challenges of Weaving the History of Mathematics into UAE Educational Framework	Jason Johnson
16:55 – 17:20	1219	9th -Grade Students' Difficulties in Solving Systems of Equations with Three Variables in Abu Dhabi High Schools	Nadeia Alalawi
17:20 – 17:45	1215	Improving Educational Accessibility for Mathematics Students in the Middle East	Rajani Rani Gupta
17:45 – 18:10		Group Discussion	

Special Session B: Applied and Computational Analysis (SS2) Thursday February 20 Chair: Amjad Tuffaha Room: CHM225			
Time	**ID**	**Title**	**Speaker**
16:30-17:10	1245	Sustained oscillations in hyperbolic-parabolic systems	Athanasios Tzavaras
17:10-17:50	1261	On Some Inverse Boundary Value Problems Arising from Cardiac Electrophysiology	Elena Beretta
17:50 – 18:30	1262	Algorithm Development for Solution of Dynamic Poroelasticity equations based on the Spectral Method	Dana Bliyeva

Special Session B: Operator Theory and Harmonic Analysis (SS6) Thursday February 20 Chair: Issam Louhichi Room: CHM113			
Time	**ID**	**Title**	**Speaker**
16:30-16:50	1052	On Quadrature-Difference Methods for Solving Periodic Linear and Nonlinear Singular Integro-Differential Equations	Alexander Fedotov
16:55-17:15	1091	Rectangular Hardy Operator in Weighted Lebesgue Spaces	Elena Ushakova
17:20-17:40	1119	Series Expansions of Solutions of Parabolic Differential-Difference Equations	Andrey Muravnik
17:45-18:05	1111	Carleson-Type Embeddings with Closed Range	Konstantin Dyakonov
18:10-18:30	1205	On the Commuting Problem of Toeplitz Operators on the Harmonic Bergman Space	Hasan Iqtaish

Special Session B: Rings, Monoids, Module Theory, and Graphs from Rings (SS7) Thursday February 20 Chair: Kuncham Syam Prasad Room: NAB007			
Time	**ID**	**Title**	**Speaker**
16:30 – 16:55	1232	Characterization Of Generalized Lie-Type Derivations on Rings and Algebras	Mohammad Ashraf
16:55 – 17:20	1025	Inverse Semigroups and Leavitt Path Algebras	John Meakin
17:20 – 17:45	1153	On Different Classes of Prime Hyperideals in Hyperlattices	Harikrishnan Panackal
17:45 – 18:10	1201	Some Commutativity Results in Nearrings	Kedukodi Babushri Srinivas Darani
18:10 – 18:35	1169	Zero-Divisor Graphs of Semirings	Omar Almaimouni

Session B: Statistics & Machine Learning (GS6) Thursday February 20 Chair: Mujo Mesanovic Room: NAB009			
Time	**ID**	**Title**	**Speaker**
16:30 – 16:50	1234	Unbiased Parameter Estimation for Partially Observed Diffusion	Elsiddig Awadelkarim
16:50 – 17:10	1117	LLM-Enhanced Deep Reinforcement Learning for Automated Trading	Rahul Tak
17:10 – 17:30	1042	Enhancing Pde Solutions with Machine Learning as Series Methods	Habib Ullah
17:30 – 17:50	1149	RPA- Driven Transformation in the UAE Motor Insurance Sector: An Assessment of Efficiency, Customer Experience, and Market Positioning	Dhivya Justin

Session B: Analysis (GS2) Thursday February 20 Chair: Mohammad Alakhrass Room: NAB 010			
Time	**ID**	**Title**	**Speaker**
16:30 – 16:50	1083	Quasi Hyperharmonic Functions in Axiomatic Harmonic Spaces.	Mohammad Alakhrass
16:50 – 17:10	1037	Generalized Mittag-Leffler-Confluent Hypergeometric Functions in Fractional Calculus with Integral Operator	Firas Ghanim
17:10 – 17:30	1200	Boundedness Of Coupled Pseudo-Differential Operators on Schwartz Space	Kanailal Mahato
17:30 – 17:50	1125	A Mathematical Analysis of a System of Caputo-Fabrizio Fractional Differential Equations for Cancer Combination Treatment	Nasrin Eghbali

DAY 2: February 21, 2025

Special Session C: Statistics and Data Science for Digital Finance on Tokenomics (SS9) Friday February 21 Chair: Stephen Chan/ Jeffrey Chu Room: NAB006			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:50	1108	Digital Assets in War - A Double-Edged Sword	Jeffrey Chu
10:50 - 11:10	1107	Stylized Facts of Metaverse Non-Fungible Tokens	Durga Chandrashekhar
11:10 - 11:30	1203	Ethereum Blockchain Analytics: What We Learn on Token Price from Geometric Deep Learning Models	Yuzhou Chen
11:30 - 11:50	1109	Reaction Times to Economic News in High-Frequency Trading: An Analysis of Latency and Informed Trading Ahead of Macro-News Announcements	Joerg Osterrieder
11:50 - 12:10	1074	Predicting Financial Trends Using Text Mining and NLP	Fulvio Raddi

Special Session C: Algebraic Topology (SS1) Chair: Pavle Blagojevic Friday February 21 Room: CHM227			
Time	**ID**	**Title**	**Speaker**
10:30 - 11:20	1253	Applications of Homotopy Theory in Physics	Hisham Sati
11:30 - 12:20	1230	Homotopy Theory for Topological Quantum Computing	Urs Schreiber

Special Session C: Applied and Computational Analysis (SS2) Friday February 21 Chair: Edriss Titi Room: CHM225			
Time	**ID**	**Title**	**Speaker**
10:30 - 11:10	1258	Enhancing Red Sea Forecasting: Integrating Data Assimilation, Uncertainty Quantification, and Artificial Intelligence	Ibrahim Hoteit
11:10 - 11:50	1270	On the desingularization of time-periodic vortex motion in bounded domains	Zineb Hassainia

Special Session C: History of Mathematics (SS3) Friday February 21 Chair: Saadia Khouyibaba Room: NAB008			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:55	1245	The Origin of Bicomplex Numbers in Cockle's Tesserines	Daniele Struppa
10:55 - 11:20	1115	Algebraic Accounts of Division and Ratio Focus on Some Arabic Sources	Eleonora Sammarch
11:20 - 11:45	1061	The Mathematics of the Astrolabe and its History	Graziano Gentili
11:45 - 12:10	1062	Experience and Knowledge in the 17th Century, Royal Society Scientists and IBN Tufayl	Luisa Simonutti
12:10 - 12:35	1270	Exploration of al Khwarizmi's work on Quadratic Equations	Saadia Khouyibaba

Special Session C: Operator Theory and Harmonic Analysis (SS6) Friday February 21 Chair: Fernando Leon-Saavedra Room: CHM 113			
Time	**ID**	**Title**	**Speaker**
10:30-10:50	1097	Algebraic Properties of Dimovski Spaces for Operational Calculus	Arran Fernandez
10:55-11:15	1217	A Partial Solution to Ricceri's Conjecture	Francisco Javier Garcia-Pacheco
11:20-11:40	1220	Frequent Hypercyclicity of Some Non-Convolution Operators	Fernando León-Saavedra
11:45-12:05	1135	Convolutions on Lie Groups, Generic Bessel Potential Spaces and Applications	Roland Duduchava
12:10-12:30	1246	Direct and Inverse Problems for the Poisson Equation with Equality of Flows on a Part of Boundary	Makhmud Sadybekov

Special Session C: Rings, Monoids, Module Theory, and Graphs from Rings (SS7) Friday February 21 Chair: Andreas Reinhart Room: NAB007			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:55	1130	Prime Ideals in Noetherian Rings	Sylvia Weigand
10:55 - 11:20	1055	Polynomial Krull Domains	Giulio Peruginelli
11:20 - 11:45	1001	Polynomial And Power Series Rings Over a Generalized Krull Ring	Gyu Whan Chang
11:45 - 12:10	1138	Homological Characterizations Of G-Krull Domains And G-Dedekind Domains	Hwankoo Kim
12:10 - 12:35	1076	Equations And Characterizations for Intermediate Rings	Ali Jaballah

Special Session C: Statistical Learning and Data Science (SS8) Friday February 21 Chair: Ayman Alzaatreh Room: CHM226			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:55	1249	Refining Bayesian Analyses with Improper Priors Using K-fold Cross-Validation	Luai Al-Labadi
10:55 - 11:20	1120	A Compositinal Data Analysis of Physical Activity Patterns of School-Aged Children in UAE	Rafiq Hijazi
11:20 - 11:45	1242	Support Vector Machine Control Chart for Multivariate Data	Mahmoud Awad
11:45 - 12:10	1020	Accommodating Spatial Outliers in Spatial Regression Model with Applications On COVID-19 Data	Ali Mohammed

Session C: Graph Theory, Coding Theory and Cryptography (GS4) Friday February 21 Chair: Carlos Fonseca Room: NAB004			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:50	1079	Minimal Weighted Acyclic Graphs	Carlos Fonseca
10:50 - 11:10	1048	Weighted Topological Index of Graphs	Zahid Raza
11:10 - 11:30	1123	Extremal Chemical Trees for Exponential Augmented Zagreb Index	Sourav Mondal
11:30 - 11:50	1122	On Szeged-Type Indices of Zeolite BCT Framework	Kavin Jacob
11:50 - 12:10	1047	Face Degree Indices of Benzinoid Hydrocarbons	Thirsha R
12:10 – 12:30	1236	ℓ-intersection Pairs of Constacyclic and Conjucyclic Codes	Ramakrishna Bandi

Special Session C: Modeling and Simulation of Biological Systems (SS5) Friday February 21 Chair: Filippo Castiglione Room: NAB005			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:55	1252	New Optimal Control Approach to Contain Epidemic in Case of Limited Resources	Abdessamad Tridane
10:55 - 11:20	1164	Agent-Based Modeling to Enhance *In Vitro* Studies of Tuberculous Granuloma Formation	Enrico Mastrostefano
11:20 - 11:45	1110	Detection of Parkinson's disease through EEG brain signals using Open- Look, Listen and Learn	Ali Aldujaili
11:45 - 12:10	1213	Stability of Threshold Boolean Networks	Hadeel Kittaneh

Session C: Applied Mathematics & Differential Equations (GS3) Friday February 21 Chair: Rima Cheaytou Room: NAB 010			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:50	1039	Error Analysis of The Vector Penalty-Projection Methods for The Time-Dependent Stokes Equations with Open Boundary Conditions	Rima Cheaytou
10:50 - 11:10	1066	Periodic Sequences Generating Equivalent Classes in The Unit Circle	Mostafa Zahri
11:10 - 11:30	1116	Solving The Fully Fuzzy Riccati Matrix Equation in Linear Quadratic Regulator Problems Using Gradient and Least Square Iterative Methods	Ahmed Elsayed
11:30 - 11:50	1070	Study On Mass Loading Sensitivity in Buffer Layered Piezo – Magnetic Fiber-Reinforced Composite Structure Analyzing Shear Wave Propagation	Aditya Kanaujiya
11:50 - 12:10	1071	Mathematical Analysis on Vibrations of Piezo-Electro-Magnet-Thermoelastic Composite Nano-Beams Under Three Phase Lag Model	Abhishek Singh
12:10 - 12:30	1113	Mathematical Analysis of Surface Wave Vibration İn FGPM Composite Structure	Sanjeev Sahu

Session C: Statistics & Machine Learning (GS6) Friday February 21 Chair: Iman Al Hashmi Room: NAB 009			
Time	**ID**	**Title**	**Speaker**
10:30 - 10:50	1034	Lomax – Pearson Type Vii Distribution with Applications to Financial Stock Returns	Iman Al Hashmi
10:50 - 11:10	1022	Towards Robust Prediction Using the Elliptical Process for Regression	Abir Al Khabori
11:10 - 11:30	1008	The Impact of Audit and Reporting Standards on The Dynamic Model of Foreign Portfolio Investment	Ioana Vinasi
11:30 - 11:50	1049	Estimation For Conditional Moment Models Based on Martingale Difference Divergence	Kunyang Song
11:50 - 12:10	1050	Generation Of Non-Linear Synthetic Time Series of EEG Data Using Lstm Models	Bakr Rashid Majeed Al Qaysi
12:10 - 13:30	1059	Improved Cusum Control Chart for Process Variance Using Bayerian Method	Tahir Abbas

Special Session D: Statistics and Data Science for Digital Finance on Tokenomics (SS9) Friday February 21 Chair: Joerg Osterrieder Room: NAB006			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:20	1056	Does the Interplay Between Pollution and Economic Connectivity Impact Happiness? Evidence from European Countries	Alexandra-Anca Purcel
14:20 - 14:40	1139	Digital Innovations' Contribution to Reducing the Shadow Economy in Emerging EU Countries	Adrian Bojan
14:40 - 15:00	1142	The Impact of Corporate Governance on the Involvement in Corporate Social Responsibility Activities	Ruben Pop

Special Session D: Algebraic Topology (SS1) Chair: Andres Angel Friday February 21 Room: CHM227			
Time	**ID**	**Title**	**Speaker**
15:00 - 15:50	4003	Cubes, Comonads and Calculus	Kathryn Hess
15:50 - 16:50	1176	Mapping Curves to Blowups	Ronno Das

Special Session D: Operator Theory and Harmonic Analysis (SS6) Friday February 21 Chair: Abdel Rahman Yousef Room: CHM 113			
Time	**ID**	**Title**	**Speaker**
14:00-14:20	1240	Improved Hardy Inequalities	Durvudkhan Suragan
14:25-14:15	1126	Slow Propagation Velocities in Schrödinger Operators with Large Periodic Potential	Houssam Abdul-Rahman
14:50-15:10	1247	On the Well-Posedness of the Local Boundary Vale Problem for the Involutory Elliptic Equation	Allaberen Ashyralyev
15:15-15:35	1134	Almost Everywhere Convergence of Subsequences of Partial Sums of Fourier Series with Respect to Vilenkin Systems	George Tephnadze
15:40-16:00	1096	A Fractional Laplacian and its Extension Problem	Salem Ben Said

Special Session D: Rings, Monoids, Module Theory, and Graphs from Rings (SS7) Friday February 21 Chair: Giulio Peruginelli Room: NAB007			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:25	1054	One Absorbing Factorization Lattices	Andreas Reinhart
14:25 - 14:50	1129	ON Phi--(2, J)-Ideals of Commutative Rings	Eda Yıldız
14:50 - 15:15	1159	Generalized The Essential Ideal Graph of an N-Group	Kuncham Syam Prasad
15:15 - 15:40	1147	Power Order Graph of A Finite Group	Saba Al-Kaseasbeh
15:40 - 16:05	1099	The N-Total Graph of a Commutative Ring	Djamila Ait Elhadi

Special Session D: Statistical Learning and Data Science (SS8) Friday February 21 Chair: Luai Al-Labadi Room: CHM226			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:25	1082	Estimation of Multiple Linear Regression Model Parameters under Multicollinearity: A Comparative Study of New Ridge-Type Pretest and Shrinkage Estimators with Machine Learning Approaches. An Application to Marsh Grass Biomass.	Marwan Al-Momani
14:25 - 14:50	1133	A Non-Invasive Diagnostic Tool and Risk Calculator for Accurate Diagnosis of Thyroid Disorders	Areej Mohammed
14:50 - 15:15	1155	Lifecycle-Based Enhancement of Physical Asset Management in Utilities	Alya Alquraidi
15:15 - 15:40	1221	Relative Range Sequential Approach for Detecting Outliers	Dania Dallah

Special Session D: Applied and Computational Analysis (SS2) Friday February 21 Chair: Belkacem Said Houari Room: CHM225			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:40	1003	On a Thermoelastic Bresse System Free of the Second Spectrum: Existence and Stability	Salim Messaoudi
14:40 - 15:20	1257	Unification theory of periodic solutions to active scalar equations	Taoufik Hmidi
15:20 - 16:00	1260	PiTSBiCG Parallel in time Stable Bi-Conjugate Gradient method	Mohamed Riahi

Special Session D: Modeling and Simulation of Biological Systems (SS5) Friday February 21 Chair: Abdessamad Tridane Room: NAB005			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:25	1157	Modelling the Fear Factor as Delay Spatiotemporal Epidemic Model	Ghilmana Sarmad
14:25 - 14:50	1243	Dynamics of HIV-1 and DC4+ T Cells Interactions Via Pseudospectrum	Ksenija Doroslovacki
14:50 - 15:15	1167	Existence Chaos and Shrimps Like Periodic Islands in ISO-Spikes Biparametric Diagrams of a Tritrophic	Anupam Priyadarshi
15:15 - 15:40	1271	Leveraging Differential Expression Network Analysis to Address Key Questions in Cancer Immunotherapy	Ayham Zaitouny
15:40 - 16:05	1194	Numerical Simulation of Lung Tissue Freezing During Cryosurgery Using TPL Bioheat Model	Sushil Kumar

Session D: Computational Mathematics and Numerical Analysis (GS5) Friday February 21 Chair: Aziz Takhirov Room: NAB 008			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:20	1057	Pressure Poisson Splitting Scheme for Navier-Stokes Equations with Open Boundaries	Aziz Takhirov
14:20 - 14:40	1166	Block Generalized Minimal Residual Method	Sofia Sukmaniuk
14:40 - 15:00	1185	On The Improving of Tt-Cross Approximation Method	Dmitry Zheltkov
15:00 - 15:20	1186	Bayesian Anomaly Detection in Variable-Order and Variable-Diffusivity Fractional Diffusion Mediums	Hamza Ruzayqat
15:20 - 15:40	1005	Advancing Convolution Quadrature Methods for Numerical Stability and Theoretical Analysis In Differential Equations	Fadi Awawdeh
15:40 - 16:00	1124	Fundamental Solution-Based Network for Solving the Inverse Cauchy Problem of Homogeneous Differential Equations	Wen Li

Session D: Applied Mathematics & Differential Equations (GS3) Friday February 21 Chair: Ljiljana Cvetkovic Room: NAB 010			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:20	1162	On The Euclidean Distance to Instability	Ljiljana Cvetkovic
14:20 - 14:40	1183	Predicting The Position of Matrix Pseudospectrum with Respect to A Given Complex Domain	Ljiljana Cvetkovic
14:40 - 15:00	1181	Optimizing Permeability Domain for Hydrogen Fuel Cells	Afrah Alzahrani
15:00 - 15:20	1184	Mathematical And Physical Aspects for The Solution Of The Initial-Value Problem For The System Describing Linear Thermodiffusion In The Thermal Stresses Theory	Jerzy Gawinecki
15:20 - 15:40	1225	Discrete-Time Replicator Dynamical Systems on Wardrop-Schur Optimal Transportation Networks	Armen Bagdasaryan
15:40 - 16:00	1226	The Price of Cognition and Evolutionary Dynamics Of Learning In Neural Networks Through Wardrop's Equilibria Approaches	Armen Bagdasaryan

Session D: Statistics & Machine Learning (GS6) Friday February 21 Chair: Rachid Bentoumi Room: NAB 009			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:20	1141	Exploring A New Theoretical Scheme to Construct Simple Asymmetric Bivariate Copulas: Application To Abu Dhabi Climatological Data	Rachid Bentoumi
14:20 - 14:40	1143	The Impact of Corporate Governance on The Involvement in Corporate Social Responsibility Activities"	Beatrice Larisa Dumnici
14:40 - 15:00	1144	Poisson-Charlier Polynomial Distribution and Its Applications	Yufan Feng
15:00 - 15:20	1152	Optimal Control of The False Discovery Exceedance in Large-Scale Directional Multiple Testing	Guozhu Tang
15:20 - 15:40	1175	The Background Stress Estimated from Stress Rotation Using a Bayesian Approach	Takaki Iwata
15:40 - 16:00	1223	A New Feature Selection Method for Qualitative Variables	Lama Sayegh

Session D: Graph Theory, Coding Theory and Cryptography (GS4) Friday February 21 Chair: Maheshanand Bhaintwal Room: NAB 004			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:20	1102	Locally Maximal Recoverable Codes	Maheshanand Bhaintwal
14:20 - 14:40	1170	Quantum Private Information Retrieval from Repairing Reed-Solomon Codes	Ramakrishna Bandi
14:40 - 15:00	1043	Comparative Analysis of Topology and Entropy Characterization in Aminal Linked Covalent Organic Frameworks and Graph Energy Prediction	Aravindan Maaran
15:00 - 15:20	1044	Mathematical Techniques for Graph Descriptors, Entropies, Spectra, And Properties of Oxalate-Based Metal Organic Frameworks	Arokiya Doss C.I.
15:20 - 15:40	1045	Comparative Studies of Topological Indices of Chemical Structures	Celin Fiona J
15:40 - 16:00	1046	Topological Indices, Graph Entropies and Regression Models of Covalent Organic Frameworks	Jency Jose

Sponsor Session E Chair: Saadia Khouyibaba Friday February 21 Room: CHM226			
Time	**ID**	**Title**	**Speaker**
16:30 – 5:10	3003	McGraw Hill Presentation	

Session E: Algebra, Number Theory and Applications (GS1) Friday February 21 Chair: Irakli Chitaia Room: NAB 007			
Time	**ID**	**Title**	**Speaker**
16:30 - 16:50	1095	Nonspeedable Sets and Q1-Reducibility	Irakli Chitaia
16:50 - 17:10	1100	Further Results Related to The Numerical Radius of Block Matrices	Aliaa Burqan
17:10 - 17:30	1051	Exploring Kaprekar's Algorithm in Alternative Digit Arrangements	Lujain Khalil
17:30 - 17:50	1196	2-Uniform Covering Groups of Elementary Abelian 2-Groups	Dana Saleh

Session E: Applied Mathematics and Differential Equations (GS3) Friday February 21 Chair: Mohammad Al-Refai Room: NAB 010			
Time	**ID**	**Title**	**Speaker**
16:30 - 16:50	1064	Gronwall Type Inequalities for Normalized Fractional Integrals with Mittag-Leffler Kernels with Applications	Mohammed Al-Refai
16:50 - 17:10	1238	Stability Analysis of Charged Neutron Stars and Darmois Junction Conditions	Abdur Rehman Jami
17:10 - 17:30	1158	Structure Preserving Reduced-Order Model for Parametric Cross-Diffusion Systems	Jad Dabaghi
17:30 - 17:50	1235	Role Of Temperature-Dependent Features in Wave Propagation Through Transversely Isotropic Micropolar Thermoelastic Media	Rajani Rani Gupta
17:50 - 18:10	1172	Analysis Of Numerical Solution of Functional Differential Equations	Rupal Aggarwal
18:10 - 18:30	1231	Rate Of Approximation by Generalized Sampling Operators	Sathish Kumar A

Session E: Statistics & Machine Learning (GS6) Friday February 21 Chair: Samir Safi Room: NAB 009			
Time	**ID**	**Title**	**Speaker**
16:30 - 16:50	1085	Identification And Estimation of Change Points in Factor Models for High-Dimensional Time Series Data	Xialu Liu
16:50 - 17:10	1086	Beyond Linear Regression: Enhancing Predictive Accuracy in Stock Price Prediction Using Ensemble Methods	Samir Safi
17:10 - 17:30	1092	Berry-Essen Bounds for Degenerate U-Statistics with Application to The Distance Correlation	Qi-Man Shao
17:30 - 17:50	1103	A Two-Stage Approach for Bayesian Network Structure Discovery Under Data Uncertainty	Hyeongmin Park
17:50 - 18:10	1132	Bayesian Approach for Hidden Markov Regression Model	Wonbin Jung

Special Session E: Applied and Computational Analysis (SS2) Friday February 21 Chair: Amjad Tuffaha Room: CHM225			
Time	**ID**	**Title**	**Speaker**
16:30 - 17:10	1259	Boundary Layer Separation	Tej Eddine Ghoul
17:10 - 17:50	1269	Efficient and optimally convergent discontinuous finite element approach on general meshes	Aymen Laadhari

Special Session E: Algebraic Topology (SS1) Friday February 21 Chair: Andres Angel Room: CHM227			
Time	**ID**	**Title**	**Speaker**
16:30 - 17:20	1014	A General Notion of Coherent Systems	Alexander H.W. Schmitt
17:30 – 18:00	1204	An Approach of Derived (Co) Limits to Homology Theories	Nikita Goloub

DAY 3: February 22, 2022

Special Session F: Algebraic Topology (SS1) Saturday February 22 Chair: Ronno Das Room: CHM227			
Time	**ID**	**Title**	**Speaker**
9:10 – 10:00	1088	Torsion in the Group of Self-Homotopy Equivalences of a CW-Complex	Mahmoud Ben Khalifa

Special Session F: Applied and Computational Analysis (SS2) Saturday February 22 Chair: Amjad Tuffaha Room: CHM225			
Time	**ID**	**Title**	**Speaker**
9:00 - 9:40	1263	Well-Posedness, Absolute Boundary Stabilization and Numerical Results for a Nonlinear Timoshenko-Ehrenfest Type System	Abdelaziz Soufyane

Special Session F: Applied and Computational Analysis (SS2) Saturday February 22 Chair: Edriss Titi Room: CHM225			
Time	**ID**	**Title**	**Speaker**
10:30 – 11:10	1266	On systems of reaction–diffusion equations with a balance law: the sequel	Mokhtar Kirane
11:10 – 11:50	1206	Timoshenko Energy decay rate with Physics-Informed Neural Networks (PINNs)	Makram Hammouda

Special Session G: Algebraic Topology (SS1) Saturday February 22 Chair: Ronno Das Room: CHM227			
Time	**ID**	**Title**	**Speaker**
10:30 - 11:00	1178	Classification of COVID-19 via Homology of CT-SCAN	Muhammad Imran Qureshi
11:00 - 11:30	1233	Topologically Autoencoding Cognitive Maps	Maxim Beketov
11:45 - 12:15	1227	Persistent Cobordism (Topological Characteristic of Networks Dissimilarity).	Konstantin Sorokin

Sponsor Session: H Saturday February 22 Chair: Saadia Khouyibaba Room: CHM226			
Time	**ID**	**Title**	**Speaker**
12:00 – 12:20	3001	Wiley Presentation	

Special Session H: Applied and Computational Analysis (SS2) Saturday February 22 Chair: Abdelaziz Soufyane Room: CHM225			
Time	**ID**	**Title**	**Speaker**
14:00 - 14:40	1265	TBA	Nader Masmoudi
14:40 - 15:20	1268	On the Westervelt - Pennes Model of Nonlinear Thermo-Acoustics	Belkacem Said Houari
15:20 – 16:00	1256	Weighted Sobolev Estimates for Incompressible Fluid Interface	Omar Lazar

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THE FOURTH INTERNATIONAL CONFERENCE ON MATHEMATICS & STATISTICS (AUS-ICMS25)

Keynote Speaker: Prof. Kathryn Hess Bellwald

Title: Topological insights in neuroscience

Abstract:

The brain of each of us is composed of hundreds of billions of neurons (or nerve cells,) linked by hundreds of trillions of synapses, which transmit electrical signals from one neuron to another. In response to a stimulus, waves of electrical activity pass through the network of neurons, processing the incoming signal. Tools provided by the field of mathematics known as algebraic topology enable us to detect and describe the rich structure hidden in this dynamic tapestry.

In this talk, I'll take you on a mathematical mystery tour of what tools from topology reveals about how the brain processes information, based on collaborations with the Blue Brain Project at EPFL.

THE FOURTH INTERNATIONAL CONFERENCE ON MATHEMATICS & STATISTICS (AUS-ICMS25)

Keynote Speaker: Prof. Martin Bohner

Title: Generalized Periodicity

Abstract:

Starting with the Beverton-Holt difference equation and with the famous Cushing-Henson conjectures for a periodic environment, we begin our journey in the discrete case, continue to the time scales case, and with a detour to the quantum case, we arrive at the isolated time scales case, then return to the continuous case and end up with a generalization of the classical concept of periodic functions.

THE FOURTH INTERNATIONAL CONFERENCE ON MATHEMATICS & STATISTICS (AUS-ICMS25)

Keynote Speaker: Prof. Edriss Titi

Title: Rigorous Analysis and Numerical Implementation of Nudging Data Assimilation Algorithms

Abstract:

In this talk, we will introduce downscaling data assimilation algorithms for weather and climate prediction based on discrete coarse spatial scale measurements of the state variables (or only part of them, depending on the underlying model). The algorithm is based on linear nudging of **the coarse spatial scales** in the algorithm's solution toward the coarse spatial scales corresponding to the observed measurements of the unknown reference solution. The algorithm's solution can be initialized arbitrary and is shown to converge at an exponential rate toward the exact unknown reference solution. This indicates that the dynamics of the algorithm is globally stable (not chaotic) unlike the dynamics of the model that governs the unknown reference solution. Capitalizing on this fact, we will also demonstrate uniform in time error estimates of the numerical discretization of these algorithms, which makes them reliable upon implementation computationally. Furthermore, we will also present a recent improvement of this algorithm by employing nonlinear nudging, which yields super exponential convergence rate toward the unknown exact reference solution. Notably, this algorithm applies to all dissipative systems, however, we will show by examples that it does not work for non-disspative systems.

SPECIAL SESSIONS

SS1 - Algebraic Topology and Applications

Pseudo-Anosov Maps and Integer Permutations

John Hubbard, **Ahmad Rafiqi** [1], Tom Schang

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Abstract: We provide a characterization of pseudo-Anosov maps in terms of integer permutations. We show that an orientation-preserving pseudo-Anosov homeomorphism with orientable foliations and fixing all critical trajectories can be encoded as a permutation of $2g + \nu - 1$ positive integers, where ν is the number of distinct singularities on the surface of genus g. The permutation satisfies an ADMISSIBILITY condition. Conversely, we show that the pseudo-Anosov map can be uniquely constructed out of such data. In particular, for closed oriented surfaces, we construct in this way every orientable foliation invariant under a pseudo-Anosov homeomorphism.

We further show how this data is especially amenable to computing the Teichmüller polynomial of the associated fibered face of the hyperbolic 3-manifold that arises as the mapping torus of the homeomorphism.

Keywords: pseudo-Anosov, permutation, entropy

2010 Mathematics Subject Classification: 37B40, 54C70, 55R10

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A GENERAL NOTION OF COHERENT SYSTEMS

Alexander H.W. Schmitt

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Abstract: Bradlow, Brambila-Paz, García-Prada, and Gothen suggested to study coherent systems for Higgs bundles in order to get a better understanding of the geometry of moduli spaces of Higgs bundles. We will look at a wider class of coherent systems for decorated vector bundles and propose a notion of semistability which fits into the general framework of Kobayashi–Hitchin correspondences [1]. In the special case of tensor powers, we will study this notion more closely by doing some non-trivial constructions and computations in geometric invariant theory. It is an interesting aspect that ampleness of the linearization in the geometric invariant theory construction yields a bound on the stability parameter for coherent systems. We will report on our work in [6] and [7] which builds on the papers [3] and [4] by King/Newstead and Le Potier, respectively, which contain the fundamentals of the theory of classical coherent systems, relevant, e.g., for Brill–Noether theory of vector bundles, and general techniques for dealing with decorated vector bundles from [5]. In addition, we will mention recent work of Cesare Goretti on the Kobayashi–Hitchin correspondence [2].

Keywords: Coherent system, moduli space, geometric invariant theory.

2010 Mathematics Subject Classification: 14H60, 14D20, 14L24.

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Equivariant bordism in dimension 2 and the evenness conjecture of equivariant unitary bordism

Andres Angel, Eric Samperton, Carlos Segovia and Bernardo Uribe

Los Andes University (Colombia), UNIANDES *ja.angel908@uniandes.edu.co*

Abstract: or a finite group, the equivariant bordism ring is a module over the usual (non-equivariant) cobordism ring. For G abelian or metacyclic, the equivariant unitary bordism ring is a free module in even degrees. It was conjectured that this should be true for any general group. We showed that the conjecture is false by finding explicit counterexamples and explicitly described the 2-dimensional equivariant unitary and oriented bordism groups for all finite groups. This talk is based on joint work with Eric Samperton, Carlos Segovia and Bernardo Uribe.

Applications of Homotopy Theory in Physics

Hisham Sati

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Abstract:

I will survey how concepts and constructions from homotopy theory enters in the description of various fundamental features of M-theory and string theory.

CUBES, COMONADS, AND CALCULUS

Kathryn Hess, Brenda Johnson, Kristine Bauer, Robyn Brooks, Julie Rasmusen, Bridget Schreiner

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General area of research: Topology and Geometry

Abstract: Abstracting the framework common to most flavors of functor calculus, one can define a calculus on a category M equipped with a distinguished class of weak equivalences to be a functor that associates to each object x of M a tower of objects in M that are increasingly good approximations to x, in some well defined, Taylor-type sense. This definition dualizes in an obvious sense, giving rise to the notion of a cocalculus. Such (co)calculi can be applied, for example, to testing whether morphisms in M are weak equivalences.

In this talk, after making the definition above precise, I will describe machines for creating (co)calculi on functor categories Fun (C,M), naturally in both the source C and the target M. The naturality of this construction makes it possible to compare both different types calculi on the same functor category, as well as the same type of calculus on different functor categories. I will briefly sketch a few examples.

The key mechanism in the calculus machine is the natural construction of a comonad on a functor category Fun (D,M) from a cubical family of commuting localizations of D, and dually for the cocalculus machine.

PERSISTENT COBORDISM (TOPOLOGICAL CHARACTERISTIC OF NETWORKS DISSIMILARITY)

Konstnatin Sorokin, Aleksandr Levin

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Abstract:

We propose a novel approach to distinguishing the networks of the same global properties (say, core-periphery structure, small-worldness, hyperbolicity, etc.), but differing drastically locally. A good example of such a network is a functional connectome [1] in a brain, which is evolving over time while an animal is learning something, a topology of an ambient space, for example [2]. The proposed approach is not a metric, but rather a (topological) characteristic of the network evolution, say when it changes dynamically along some timeline. A simple universal example of such a networks are multiplexes. Our approach is well-suited to describe their rate of changing over time. At the same time we build a generalization of that — a filtration of long exact sequences of paired simplicial complexes obtained from changing point clouds typical in topological data analysis. Indeed, for each point cloud of some data at each moment of time we build a filtration of simplicial complexes as the nerves of the coverings by Alexandrov [3] and then build our proposed construction pairing them as generalized multiplexes. After that homotopically equivalent parts of pairs of complexes can be factorized automatically by an algorithm. The remaining part could be analyzed with Mayer-Vietoris long exact sequence. This gives a topological characteristic of metric distortion of data at each step. After that for each time step of a persistence diagrams of that construction we can build corresponding long exact sequence and build decreasing filtration of each member (overall $n-1$ steps). We called this filtration of long exact sequences a "Persistent Cobordism" as the cobordism was defined in case of a PL-manifold [4].

Keywords: Topological Data Analysis, Cobordism, Multiplex Networks

2010 Mathematics Subject Classification: 55-06 Algebraic topology

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TOPOLOGICALLY AUTOENCODING COGNITIVE MAPS

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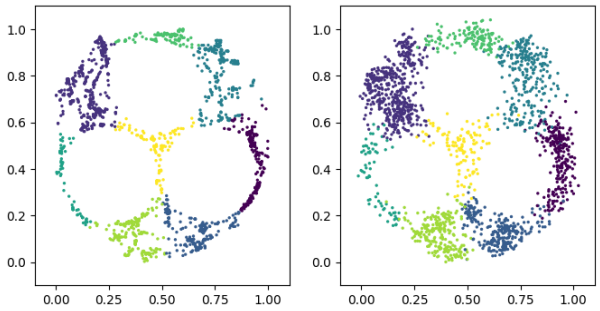
Abstract:



Left: Mouse's true coordinates (normalized), binned by proximity. *Right*: Mouse's coordinates (normalized) decoded from the neural activity of its 300 place cells with a 3-layer AutoEncoder.

Cognitive maps is a term used to describe the inner mental representation of various spaces, including the physical space – perceived by the animal present in it (or imagined, for species capable of so). These maps are encoded in the neural activity of so-called place cells (in particular – other similar neurons have also been found) – neurons found in a certain region of the hippocampus, the discovery of which has been acknowledged by a Nobel Prize in Physiology and Medicine 2014, awarded to John O'Keefe, May-Britt Moser and Edvard I. Moser [1]. In this work, we apply AutoEncoders (AEs, a family of ANNs) to the place cells' neural activity data recorded (with the optical technique of calcium imaging) from mice placed into arenas of different topologies – ones with 0, 1, 2, 3 holes (impenetrable obstacles) in it. The goal is, having the true coordinates of mice locations (synced with their neural activity) – to reconstruct this "true map" from the high-dimensional neural activity time series, preserving the topology of the arena. Applying vanilla AEs to this problem in a supervised manner (additionally penalising for metric dissimilarity of the inner, latent, representation of the time-series from the true map), we find that mices' cognitive maps are well reconstructed by quite shallow (3-4 layers) AEs – which provides some empirical implications for neuroscience. We also try a novel technique of Topological AutoEncoders (TopoAE, [2]) to solve this problem in an unsupervised manner – the TopoAE is not being shown the true coordinates – rather being penalised for a difference in topological properties (persistent homologies) of the input point cloud and its latent representation. We slightly modify this approach to have different regularisation strengths for homologies of different order – in our problem, 1-homologies (cycles) are more important than others – and overall demonstrate the success of applying such nonlinear topologically-regularised dimension reduction techniques to such neural data.

Keywords: topological data analysis, cognitive neuroscience, machine learning.

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AN APPROACH OF DERIVED (CO)LIMITS TO HOMOLOGY THEORIES

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Abstract:

In our talk, we shall dive into a number of phenomena related to the derived (or higher) (co)limit approach to the description of functors of homological nature. This approach was initially exemplified in Quillen's work on Hochschild homology of algebras [1], where he showed that Hochschild homology of an algebra can be represented as limits of some functors from the category of free extensions of a given algebra to the category of abelian groups. By examining these and related phenomena, we aim to elucidate the broader implications of this approach within homological algebra.

Further similar results were derived by Roman Mikhailov and Sergey Ivanov in the context of group homology [6], [7], [4], who showed that homology of a group G can be expressed as higher limits and colimits of certain functors along the category of free extensions $\mathsf{Pres}(G)$ of G consisting of short exact sequences $1 \to R \to F \to G \to 1$ with F being free. For instance they derived the following:

$$\mathsf{colim}_n F_{ab} \simeq H_{n+1}(G);\ \mathsf{lim}^i(R_{ab}^{\otimes n})_G \simeq H_{2n-i}(G), 0 \leq i < n,$$

where $R_{ab} = \frac{R}{[R,R]}$ is a relation G-module with action of G defined by conjugation and $\mathsf{lim}^i, \mathsf{colim}_n$ are the derived limit and colimit functors respectively.

They further proposed the construction of fr-language, where f, r are functorial ideals $f = ker(\mathbb{Z}F \to \mathbb{Z})$, $r = ker(\mathbb{Z}F \to \mathbb{Z}G)$ of the functor of group rings $\mathbb{Z}[-] : 1 \to R \to F \to G \to 1 \mapsto \mathbb{Z}F$. It allows us to describe lots of graded functors from the category of groups to the category of abelian groups by simple expressions of the form $fr \cap rf, rr + fff, \ldots$ and in particular there are isomorphisms:

$$\mathsf{lim}^1(fr^n + r^n f) \simeq H_{2n}(G),\ \mathsf{lim}^1(rr + fff) \simeq \mathsf{Tor}(G_{ab}, G_{ab}).$$

In our work recent work [8], we have provided a formalisation of their construction and defined new languages using the lower central series, which have a close connection with nilpotency. For instance we give the following higher limit description of the group homology functors with coefficients in cyclic groups:

$$\mathsf{lim}^i(\wedge^{rp} R_{ab})_G \otimes \mathbb{Z}_{(p)} \simeq H_{r(p+2)-i}(G; \mathbb{Z}/p\mathbb{Z}), i = 0, 1,$$

$$\mathsf{lim}(\wedge^{p^2} R_{ab})_G \otimes \mathbb{Z}_{(p)} \simeq H_{p^2+2}(G; \mathbb{Z}/p\mathbb{Z}) \oplus H_{p^2+2p}(G; \mathbb{Z}/p\mathbb{Z}),$$

where G is any group with no nontrivial elements of torsion $\leq p^2$, p is prime and $r < p$. Considering the example of Hochschild and cyclic homology of algebras one may notice that the derived (co)limit allows not only give another description for certain homological functors but to define homology theories themselves! We shall argue that the higher (co)limit approach may be treated as a unifying framework for defining homology theories.

We shall demonstrate how the higher (co)limit approach can be formulated within any homotopy theory using ∞-categories of fibrant n-cubes of cofibrant extensions. This framework has interesting connections with the the combinatorial description of homotopy groups of suspension spaces (e.g. spheres) [10].

In that regard we argue how considering the algebraic K-theory as a homology theory of Waldhausen categories [9] we may describe it via the derived colimit approach in the homotopy theory of Waldhausen categories.

Keywords: Derived (co)limits, algebraic K-theory, group homology.

2010 Mathematics Subject Classification: 18Gxx

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Hochschild-Serre Spectral Sequence for Lie Conformal Algebras

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Abstract:

Lie conformal algebras, originally introduced by Kac, encode an axiomatic description of the operator product expansion of chiral fields in conformal field theory. In particular, Lie conformal algebras provide a powerful tool for studying the infinite-dimensional Lie algebras and associative algebras satisfying the locality property. The most important examples of Lie conformal algebra include the Virasoro algebra Vir, the current algebra $\operatorname{Cur}\mathfrak{g}$, and their semidirect product $\mathrm{Vir} \ltimes \operatorname{Cur}\mathfrak{g}$.

In this work, we construct the Hochschild–Serre spectral sequence for Lie conformal algebras in which we follow a similar approach to the original work done by G. Hochschild and J-P. Serre for the case of Lie algebras. As applications of this construction, we calculate the cohomology of the semidirect product $\mathrm{Vir} \ltimes \operatorname{Cur}\mathfrak{g}$ with trivial coefficients and give an explicit computations for the basic cohomology of $\mathrm{Vir} \ltimes \operatorname{Cur}\mathfrak{g}$ with coefficients in its finite conformal modules $M_{\Delta,\alpha,U}$.

Keywords: Lie conformal algebras, reduced cohomology, basic cohomology.

2010 Mathematics Subject Classification: Primary 17B69, Secondary 17B63; 17B56.

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MANY PARTITIONS OF MASS ASSIGNMENTS

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General area of research: Topology and Geometry

Abstract: Problems of the existence of mass partitions by affine hyperplanes in a Euclidean space have a long and exciting history since the 1930's ham-sandwich theorem of Hugo Steinhaus and Karol Borsuk.

In this lecture, we will discuss the most general problem of partitioning families of masses, the so-called mass assignments, by chosen classes of arrangements of hyperplanes over Euclidean vector bundles. Developing a new configuration test map scheme, as well as an alternative topological framework, we are able to reprove known results, extend them to arbitrary bundles as well as put various types of constraints on the solutions. Moreover, the developed topological methods allow us to give new arguments and extend results of Guth & Katz, Schnider and Soberon & Takahashi. In this way we place all these results under the same "roof".

This lecture is dedicated to the memory of Frederick R. Cohen, an exceptional mathematician and an amazing human being.

Keywords: Mass partitions, parametrized Borsuk—Ulam type results, characteristic classes.

2010 Mathematics Subject Classification: 52A35 (primary); 57R20, 55N25 (secondary).

Mapping curves to blowups

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Abstract: Following Segal's work on rational functions and its many extensions, we show that the space of holomorphic maps from Riemann surfaces to certain blowups of projective space has homological stability as the degree gets large. We will also explore arithmetic applications to the Batyrev-Manin conjecture for del Pezzo surfaces, by applying the Grothendieck-Lefschetz trace formula. The talk will be based on joint work with Brian Lehmann, Sho Tanimoto and Philip Tosteson.

Homotopy Theory for Topological Quantum Computing

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Abstract:

While topological quantum hardware is expected to be controlled by the mathematics of braided fusion categories, little attention has been paid to a more first-principles understanding of this phenomenon. We will survey recent advances [1][2][3] via "geometric engineering" that make crucial use of geometric homotopy theory for "flux quantization" [4].

Talk notes available at: ncatlab.org/schreiber/show/ICMS2025

Keywords: configuration spaces, Pontrjagin homology algebra, braid representations

2010 Mathematics Subject Classification: 55R80, 57R56, 57K16, 55Q55, 55925, 55N32

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TORSION IN THE GROUP OF SELF-HOMOTOPY EQUIVALENCES OF A CW-COMPLEX

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Abstract:

Let X be a simply connected CW-complex of dimension n. This talk aims to establish a link between the groups $\mathcal{E}(X)$ and $\Gamma_n(X)$. Here, $\mathcal{E}(X)$ represents the group of self-homotopy equivalences of X, while $\Gamma_n(X)$ denotes the group appearing in Whitehead's certain exact sequence of X. This pursuit provides significant insights into discerning the presence of torsion elements within $\mathcal{E}(X)$.

Keywords: Whitehead's exact sequence, Group of self-homotopy equivalences

2010 Mathematics Subject Classification: 55P10

Classification of COVID-19 via Homology of CT-SCAN

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General area of research: Applied Algebraic Topology

Abstract: The automated analysis of biomedical images plays a vital role in facilitating early diagnosis, particularly for lung diseases such as COVID-19. Early detection enables timely treatment, reduces complications, and improves patient outcomes. Given the global burden of COVID-19, with redover 776.7 million cases reported, reliable automated diagnostic techniques are essential for swift case identification and intervention. Traditional methods, such as RT-PCR, can produce false negatives, while chest CT-scans analyzed by radiologists are more sensitive but time-consuming.

Artificial Intelligence (AI) has significantly advanced medical imaging, but deep learning models often require large datasets and computational resources. In contrast, Topological Data Analysis (TDA), particularly persistent homology (PH), provides an efficient and interpretable approach by extracting topological and geometrical features from data. PH identifies features that persist across scales, offering robust insights into medical images.

In this work, we develop a novel model to detect COVID-19 in CT-scan images using TDA. Our method is based on an intuitive and natural idea of analyzing shapes and opacities. We quantify these topological features using persistent homology, a technique from Topological Data Analysis (TDA). To ensure reproducibility and external validation we conducted experiments on two distinct datasets. Our approach achieved an overall F1 score of 99.4%, and 99.6%.

Keywords: COVID-19 Diagnosis, Topological Data Analysis, Persistent Homology

2020 Mathematics Subject Classification: 55N31

SS2 - Applied and Computational Analysis

WELL-POSEDNESS, ABSOLUTE BOUNDARY STABILIZATION AND NUMERICAL RESULTS FOR A NONLINEAR TIMOSHENKO-EHRENFEST TYPE SYSTEM

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Abstract:

Here, we consider the well-posedness, absolute stabilization and numerical analysis problems for a Timoshenko-Ehrenfest type system with nonlinear boundary feedback controls. We prove the existence and smoothness of a unique solution using fixed point theory and semigroup theory. By employing the integral type multiplier method, we establish the absolute exponential and polynomial stability of the system. Moreover, we provide numerical simulations to illustrate the theoretical results.

Key words: Timoshenko-Ehrenfest type system, well-posedness, exponential and polynomial stability, numerical approximation.

2010 Mathematics Subject Classification: 35B40, 35G31, 35Q35.

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SUSTAINED OSCILLATIONS IN HYPERBOLIC-PARABOLIC SYSTEMS

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Abstract: The objective of the talk is to provide examples of sustained oscillations for hyperbolic-parabolic systems. This problem was motivated by work on the existence theory for viscoelasticity of Kelvin-Voigt type, which shows propagation of H^1-regularity for the deformation gradient of weak solutions for semiconvex stored energies. While weak solutions still exist for initial data in L^2, oscillations on the deformation gradient can now persist and propagate in time.

The existence of sustained oscillations in hyperbolic-parabolic system is studied systematically via examples, in two classes of systems: (i) Examples from nonlinear viscoelasticity and (ii) the compressible Navier-Stokes system with non-monotone pressures. In several space dimensions oscillatory examples are associated with lack of rank-one convexity of the stored energy. The subject naturally leads to the problem of deriving effective equations for the associated homogenization problems. This is in general a hard problem, which can be addressed for a simple model of phase transitions using ideas from the kinetic formulation for conservation laws.

Timoshenko Energy decay rate with Physics-Informed Neural Networks (PINNs)

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Abstract:

This talk delves into the numerical investigation of the Timoshenko system coupled with thermoelasticity, accounting for second sound effects. Using the Physics-Informed Neural Networks (PINNs) framework, we approximate solutions to this coupled system and analyze the accuracy of these approximations in capturing the system's asymptotic behavior. A particular focus will be placed on understanding how the stability parameter χ influences the discrete energy decay rate, offering insights into the interplay between the physical parameters and numerical stability. By combining theoretical analysis with the flexibility of PINNs, this work highlights the potential of neural network-based methods to address challenging stability and energy dynamics in coupled thermoelastic systems.

Keywords: Timochenko systems, PINNs, Stability, Long-time behavior.

2010 Mathematics Subject Classification: 35B40, 93D15, 93D20, 92B20, 65M99.

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EFFICIENT AND OPTIMALLY CONVERGENT DISCONTINUOUS FINITE ELEMENT APPROACH ON GENERAL MESHES

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Abstract: The cardiovascular system spans multiple spatial and temporal scales, making its numerical simulation highly complex. Modern scientific computing aims to enhance our understanding of this system, from subcellular mechanisms to fluid transport and organ-level structural mechanics.
Eulerian approaches have emerged as a promising framework for tackling fluid-structure interaction [**?**, **?**] and multiphysics problems [**?**, **?**, **?**]. They naturally handle stress coupling, large deformations, and structural contact without remeshing. However, ensuring mass conservation in transport remains a significant limitation of these methods.
In this presentation, we introduce preliminary results on a numerical approach within a discontinuous finite element framework for solving very simplified model problems. The method is a foundation for solving more physically relevant problems in the future. We establish consistency, discrete stability, and perform an error analysis of the solution approximation. Numerical experiments in 1D, 2D and 3D using high-order approximation spaces confirm the optimal convergence rates.

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On the Westervelt-Pennes model of nonlinear thermo-acoustics

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Abstract:

In this work, we investigate a mathematical model of nonlinear ultrasonic heating based on a coupled system of the Westervelt equation and the hyperbolic Pennes bioheat equation (Westervelt-Pennes-Cattaneo model). Using the energy method together with a fixed point argument, we prove that our model is locally well-posed and does not degenerate under a smallness assumption on the pressure data in the Westervelt equation. In addition, we perform a singular limit analysis and show that the Westervelt-Pennes-Fourier model can be seen as an approximation of the Westervelt-Pennes-Cattaneo model as the relaxation parameter tends to zero. This is done by deriving uniform bounds of the solution with respect to the relaxation parameter.

ALGORITHM DEVELOPMENT FOR SOLUTION OF DYNAMIC POROELASTICITY EQUATIONS BASED ON THE SPECTRAL METHOD

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Abstract: The world around us is full of examples of elastoporous fluid-saturated media, including soils, rocks, geological structures, various biological tissues of living organisms and humans, trees, and plants. Therefore, in recent decades, many numerical models of the process of propagation of seismic waves in fluid-saturated porous media have been advanced to solve practical problems in various fields of geophysics, biomechanics and oil and gas field development. In parallel, realistic analytic models of a porous medium, consisting of an elastically deformable matrix filled with a viscous fluid, were developed. This in turn has made it possible to explain the observed effects during seismic studies of the properties of rocks in the presence of pore fluid. In this talk we will introduce a mathematical model for the two-dimensional case. Furthermore, we will consider a finite-difference scheme that approximates it, and which realistically describes the physical processes occurring in a fluid-saturated poroelastic medium in a half-plane characterized by various physical parameters. Specifically, for the numerical solution, a finite-difference upwind scheme on a staggered (chessboard) grid was used, which made it possible to avoid strong oscillations of the values of grid functions at the nodes of the difference grid. Moreover, an effective algorithm for the numerical solution of the dynamic poroelasticity problem has been developed in the high-performance Julia language. In addition, a software package for a simulation model has been adopted that describes the physical processes and properties of poroelastic media with visualization corresponding to experimental observations. Furthermore, an automated complex of computer programs with an interface for visualizing the results of parallel computing processes has also been employed.

ON SOME INVERSE BOUNDARY VALUE PROBLEMS ARISING FROM CARDIAC ELECTROPHYSIOLOGY

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Abstract:

Detecting ischemic regions is crucial for preventing lethal ventricular ischemic tachycardia and fibrillation. This is typically done by recording the heart's electrical activity using either noninvasive or minimally invasive methods, such as body surface or intracardiac measurements. Mathematical and numerical models of cardiac electrophysiology can provide insight into how electrical measurements can be used to detect ischemia. The goal is to combine boundary measurements of potentials with a mathematical model of the heart's electrical activity to identify the position, shape, and size of ischemia and/or infarctions. Ischemic regions can be modeled using the monodomain model, a semilinear reaction-diffusion parabolic PDE coupled with a nonlinear ODE, that describes cardiac electrical activity comprehensively. In this talk, I will focus on the case of an insulated heart without coupling to the torso. I will first review some results related to reconstructing ischemia modelled as perfect insulators for the stationary model, and then present some results obtained recently in the case of the time-dependent nonlinear monodomain mode (a semilinear parabolic equation coupled with a nonlinear ordinary differential equation) for different types of nonlinearities . Finally, I will present some new challenging inverse problems of interest for applications.

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ENHANCING RED SEA FORECASTING: INTEGRATING DATA ASSIMILATION, UNCERTAINTY QUANTIFICATION, AND ARTIFICIAL INTELLIGENCE

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Abstract: The talk will present our ongoing efforts to develop the next generation operational system for the Red Sea, aligning with Aramco's commitment to the fourth industrial revolution. This comprehensive system integrates state-of-the-art ocean-atmosphere-wave general circulation models that have been specifically developed for the Red Sea region. It is now fully operational, routinely running on the KAUST supercomputer Shaheen. I will demonstrate our real-time visualization and analytics tools that enable user-friendly analysis of the system's extensive datasets. The second part of the talk will outline some of our ongoing research activities aimed at further enhancing the system performance and expanding its capabilities. Specifically, I will showcase our latest advancements in three key areas: (i) enhancing the system forecasting skills through new ensemble DA schemes accounting for and providing information about the forecasts uncertainties, (ii) leveraging ensemble uncertainties in the system's outputs for both forward and inverse tracking of oil spills, and (iii) introducing new AI-based algorithms for DA and UQ.

PITSBICG PARALLEL IN TIME STABLE BI-CONJUGATE GRADIENT METHOD

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Abstract: This talk introduces the Parallel-in-Time Stable Bi-Conjugate Gradient (PiTSBiCG) algorithm, a new method for parallel-in-time (PinT) numerical simulations of time-dependent partial and ordinary differential equations. The PiTSBiCG algorithm offers an alternative to the widely known parareal method by reformulating the PinT problem algebraically and solving it using an adapted Bi-Conjugate Gradient Stabilized (BiCGStab) method. This approach aims to address challenges related to stability in PinT computations, particularly for stiff or nonlinear systems. The talk will outline the mathematical formulation of the algorithm and present numerical results that compare its performance to the parareal method. These results illustrate the potential of PiTSBiCG to provide a more stable and reliable solution framework for a range of time-dependent problems.

ON SYSTEMS OF REACTION–DIFFUSION EQUATIONS WITH A BALANCE LAW: THE SEQUEL

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Abstract: This talk is a sequel of the review paper on reaction–diffusion systems with a balance law by Martin and Pierre (1990) . Here, we present the results obtained after Martin and Pierre (1990) not only for classical systems but also for systems with either time fractional derivatives or space fractional derivatives accounting for anomalous diffusions.

WEIGHTED SOBOLEV ESTIMATES FOR INCOMPRESSIBLE FLUID INTERFACE

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Abstract: I will consider the case of two incompressible fluids with different densities and same viscosity which evolve (according to Darcy's law) in a porous media. Given any initial data which are in some weighted Sobolev space, I will show how to construct global solutions to the Muskat problem. The chosen weight is in the Muckenhoupt class and it allows to generalize all the previous global existence results.

ON A THERMOELASTIC BRESSE SYSTEM FREE OF THE SECOND SPECTRUM: EXISTENCE AND STABILITY

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Abstract: In this paper, we consider, in a bounded interval, the following one-dimensional linear truncated thermoelastic Bresse system:

$$\begin{cases} \rho_1\varphi_{tt} - k(\varphi_x + \psi + lw)_x - k_0 l(w_x - l\varphi) + \beta\theta_x = 0 \\ \qquad -b\psi_{xx} + k(\varphi_x + \psi + lw) = 0 \\ \rho_3 w_{tt} - k_0(w_x - l\varphi)_x + kl(\varphi_x + \psi + lw) = 0 \\ c\theta_t - \kappa\theta_{xx} + \beta\varphi_{tx} = 0, \end{cases}$$

together initial and boundary conditions. We establish the well posedness, using the semigroup theory and with the help of non-classical deferential operators. After that, we show that the system is exponentially stable if and only if the hyperbolic equations have the same speed of wave propagation. In the opposite case, we establish a polynomial decay.

Keywords: Truncated Bresse system, Fourier thermal law, Exponential decay, Polynomial decay
2010 Mathematics Subject Classification: 35B35, 35B40, 74F05, 93D05, 93D23

UNIFICATION THEORY OF PERIODIC SOLUTIONS TO ACTIVE SCALAR EQUATIONS

Taoufik Hmidi

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Abstract: In this talk, I will discuss time-periodic solutions in the context for active scalar equations under the assumption of a completely monotonic kernel. In particular, I will focus on the existence of rotating patch solutions close to Rankine vortices and present a general framework that unifies and extends prior results for various geophysical flow models. This presentation is based on joint work with Liutang Xue and Zhilong Xue.

BOUNDARY LAYER SEPARATION

Tej Eddine Ghoul

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Abstract: I will present a recent series of results related to the Prandlt equation.

ON THE DESINGULARIZATION OF TIME-PERIODIC VORTEX MOTION IN BOUNDED DOMAINS

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Abstract: In this talk, we will discuss vortex motions for Euler equations in planar domains. In this setting, the dynamics of a single vortex is governed by a Hamiltonian system, with most of its energy levels corresponding to time-periodic motion. We show that, under certain non-degeneracy conditions, it is possible to desingularize most of these trajectories into time-periodic concentrated vortex patches. The proof uses a Nash-Moser scheme and KAM techniques combined with complex geometry tools. Additionally, we will present a vortex duplication mechanism to generate synchronized time-periodic motion of multiple vortices. This is a joint work with Taoufik Hmidi and Emeric Roulley.

SS3 - History of Mathematics

EXPLORING AL KHWARIZMI'S WORK ON QUADRATIC EQUATIONS

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Abstract:

The development of classical algebra during the golden age of Islam is the sequence of three important events: the establishment of quadratic equation theory by Muhammad ibn Musa al-Khwarizmi (780–850), the initiation and development of algebraic notation by Abu Bakr ibn Muhammad al-Karaji (953–1029) and solving cubic equations by Omar Al Khayyam (1048–1131).

This presentation focuses on al-Khwarizmi's work on quadratic equations. We will talk about how he classified these equations and explain the methods of al-Jabr (completion) and al-Muqabala (balancing). We will conclude by exploring al-Khwarizmi's approach to solving quadratic equations, with a simple example of one type of equation.

Keywords: History of Mathematics, Algebra, Quadratic Equations.

2010 Mathematics Subject Classification:

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THE ORIGIN OF BICOMPLEX NUMBERS IN COCKLE'S TESSARINES

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Abstract:

The algebra of *bicomplex numbers* has a rather strange history. While it is usually acknowledged that its study began with the 1892 work of Corrado Segre, the theory has a precedent in the rather odd invention of *tessarines*, at the hand of the English mathematician James Cockle, who first wrote about them in 1848. Cockle was not a very good mathematician, and his introduction of the *tessarines* (whose algebra is isomorphic to the algebra of *bicomplex numbers*) is an amusing comedy of errors. In this talk I will discuss what was the original goal of Cockle, and why that goal was unattainable (and unreasonable). Nevertheless, in his attempt to resolve its problem, Cockle ended up introducing an object whose study is of interest even in contemporary mathematics. The proof that, in some sense, mathematics is stronger than human incompetence. This talk is based on a forthcoming work with Marco Panza.

Keywords: History of Mathematics, Bicomplex Numbers

2010 Mathematics Subject Classification: 01A55, 30G35

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ALGEBRAIC ACCOUNTS OF DIVISION AND RATIO. FOCUS ON SOME ARABIC SOURCES

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Abstract:

The question of the status and nature of algebra in the Middle Ages is far from having a single and clear answer. In my presentation, I will address this issue by examining how division and ratio are defined and presented in a selection of Arabic texts that are representative of the development of the rules for algebraic computations. I will also consider other texts related to the reading of Euclid's *Elements* made by some Islamicate scholars. Additionally, the analysis will prompt a discussion on the nature of fractions (i.e. whether they are are considered numbers, like integers) and the procedures for finding the greatest common divisor (GCD). Spanning from the 9th to the 15th centuries, the presentation will focus on a selection of sources that are representative of the "algebrization" of division and ratio. More specifically, I will examine some excerpts from the writings of al-Khwārizmī, Abū Kāmil, al-Uqlīdīsī, al-Karajī, al-Khayyām, al-Samaw'al, al-Zanjānī, and al-Kashī.

Keywords: Medieval algebra, Arabic mathematics, division

2010 Mathematics Subject Classification:

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THE MATHEMATICS OF THE ASTROLABE AND ITS HISTORY

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Abstract: This presentation is devoted to briefly trace the scientific and cultural history of the mathematics of the astrolabe. This history is interesting from several points of view, since it intertwines mathematical developments, geographical explorations, changing worldviews, and different cultures and civilizations.
We mainly illustrate a mathematical analysis of how the Celestial Sphere can be represented on the plane: we describe the remarkable properties of the stereographic projection, and we show how these properties can be used to encode the geometry of the spherical world on a flat object, and how the astrolabe enables the user to decode them. Indeed, here is where Greek and Islamic cultures come together, to give us the mathematical foundations for the astrolabe. These foundations are the consequence of some geometrical results of Apollonius, together with later refinements, whose first traces appear in the work of the Arab astronomer and mathematician Al-Farghānī.
We finally illustrate how the mathematical techniques described can be pulled together to build a simple yet powerful instrument: we will use a step-by-step process that goes beyond the pure mechanical construction, and maintains the focus on the geometrical ideas behind the astrolabe.

Keywords: Astrolabe, Celestial Sphere, Stereographic Projections.

2010 Mathematics Subject Classification: 01-02, 51M15, 51N20

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DOI: 10.5642/jhummath.202001.07. Available at: https://scholarship.claremont.edu/jhm/vol10/iss1/7

EXPERIENCE AND KNOWLEDGE IN THE 17TH CENTURY. ROYAL SOCIETY SCIENTISTS AND IBN TUFAYL

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Abstract:

The influence of the work "*Philosophys autodidactus*. *The Epistle of Hayy ibn Yaqzan"* on the thinkers and scientists of the 17th century and Royal Society, is the subject of my contribution. The philosophical novel "*Philosophys autodidactus"* was written in Arabic by ibn Tufayl, an Arab physician and scientist, poet and philosopher who lived in 12th-century Islamic Spain. The influence of Islamic culture in Europe is a vast topic that is still little studied.
The text "*Philosophus autodidactus"* had an important circulation, translated into Latin in 1670/71, was later translated into vernacular languages; it was read - among others - by Boyle, became known to Locke and Spinoza, was appreciated by Molyneux, Leibniz and Newton, and inspired the writer Defoe and the philosopher Rousseau. Why was this philosophical novel a source of inspiration for them? It tells the story of a child's experiential and self-educational journey growing up on a deserted island. On the lush island, he learns about nature, animals and the world around him in an experiential and self-educational journey. Through the senses and the construction of methodical observation and reflection, he learns the surrounding world in depth, from the formulation of the existence of a supreme god to metaphysical formulations and abstract reasoning.
In my presentation, I intend to outline the influence this text had on thinkers such as Robert Boyle, William Molyneux, John Locke, Huygens, Leibniz, and Newton through the scholarly journals of the time, *Philosophical Transactions* (1671), and *Bibliothèque universelle et historique* (1686).

Keywords: experience, Royal Society, Boyle, Locke, Newton.

2010 Mathematics Subject Classification: 01-02

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SS4 - Mathematics Education

IMPROVİNG EDUCATİONAL ACCESSİBİLİTY OF FOUNDATION PROGRAM STUDENTS AT OMAN VİA MULTİLİNGUAL VİDEO PLATFORM

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Abstract:

Modern society is built on the foundation of education, which is essential for empowering individuals and promoting socioeconomic growth. There is a rise in the number of students who are continuing their studies by attending college in the evening, especially in the Middle East. These students face numerous challenges, from balancing work and academic responsibilities to social and family responsibilities, often resulting in physical and mental exhaustion and minimizing their performance. They must manage their time in such a way that they meet their work commitments, class schedules, and study time, leaving little room for rest or social activities. Besides this, catching up with subsequent lessons becomes even harder if they miss any of their classes due to work conditions/ location/family commitments. This will significantly hinder their academic progress and overall learning experience. Through this paper, we aim to provide a solution to support such students (GFP) by offering all video lectures and supplementary materials on a single platform. It saves their time from searching Google and YouTube for missing information and facilitates guided lectures required at their study level. By enabling easy access to the material taught in the class, this platform helps students stay on track academically even when they miss class. Additionally, by offering instructional videos in multiple languages and letting users switch between languages, this platform will overcome the language barrier and make it simpler for them to comprehend the information.

Keywords: Education, Guided learning, Work-life balance, Academic responsibilities.

2010 Mathematics Subject Classification:

- **97D40**: Technology in mathematics education
- **97U80**: Educational material and multimedia
- **97B50**: Computer-assisted instruction (CAI), e-learning, and related topics
- **97C60**: Student problems and student learning

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THE IMPACT OF INTERACTIVE DIGITAL CONTENT BASED ON THE [Tibak] MODEL IN DEVELOPING PRODUCING EDUCATIONAL AIDS SKILLS

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Abstract:

Technical advancements have a significant impact on providing lecturer with tools and devices that facilitate the delivery of information to learners, the multimedia method is one of the forms of modern education techniques. The study aims to determine the effectiveness of interactive digital content based on the (Tibak) model in developing the skills of producing educational aids among lecturer-students at P college. The researcher relied on the semi-experimental approach with two groups with pre/post measurement to demonstrate the effectiveness of the independent variable teaching method (the efficacy of interactive digital content based on the Tibak model) on the dependent variable (educational means production skills). The study sample consisted of (58) male university students specialising in P college, it was distributed during the academic semester (Tibak). The results also showed that there were statistically significant differences at the level of (0.05) between the scores of the students of the two study groups in an application, which were studied through (interactive digital content based on a model (Tibak)). The researcher recommends researching training courses for male students that show how to design digital models in courses to develop various skills.

Keywords: Digital Content, Tibak, Teaching Aids

2010 Mathematics Subject Classification: Mathematics Education

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ONLINE PLATFORMS FOR CONCEPTUAL TEACHING? THE CASE OF MYMATHLAB

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Abstract:

There is no shortage of online platforms used to teach Mathematics. Top textbook publishers are now including the platforms in combination with a printed or e-textbooks, and are marketing the platforms as a substantially helpful tool for both students and instructors.

In this talk, I give an overview of some of their strengths and weaknesses of the online platform currently used for my courses, and use data collected from the platform that highlights some of its potential risks, like acquiring procedural competence only. My hypothesis is that students' overreliance on the help tools of the platform, along with the large number of procedurally-oriented questions makes the platform useful for acquiring procedural knowledge, but not necessarily for understanding concepts.

I propose modifying the reliance on a textbook platform to the creation of stand-alone courses that involve the use of interactive videos. The features for this approach would enable students to further enhance their conceptual understanding of the content, all while providing them with the interactive engagement needed to understand content. I contend that a hybrid courses that uses interactive videos and smartphone apps would be more effective in delivering and ensuring a better and more rewarding educational experience for the students

Keywords: Conceptual Understanding, Technology in Math Education, Online Platforms

2010 Mathematics Subject Classification: 97 Mathematics Education

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UNCERTAINTIES IN MEASURING THE COURSE LEARNING OUTCOMES: FUZZY SETS APPROACH

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Abstract. Course Learning Outcomes for Mathematics or Statistics courses assume uncertainties associated with the vagueness of the concept and measurements. Although we usually measure students' academic performance in precise numbers, we understand that the levels of academic achievements and competencies are also vague in nature. In this paper, we discuss some techniques of using fuzzy sets to capture the uncertainties at various levels of achievement measurement and to use them for evaluating the degrees to which the course learning outcomes met.

Key words: CLOs, Fuzzy Set, Similarity

2010 Mathematics Subject Classification: 97D30

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A Preliminary Study on Mathematics Through a Cultural Lens: Surveying the Role of Ethnomathematics in Teaching

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This presentation aims to share a preliminary study examining the integration of ethnomathematics into educational practices through the development and dissemination of a specialized survey. Grounded in the principles of ethnomathematics, the study aims to explore how cultural and mathematical practices intersect, influencing the pedagogical approaches of educators. The research emphasizes understanding the extent to which teachers are aware of and incorporate ethnomathematical perspectives in their instruction, particularly in diverse classroom

settings. The study will employ a quantitative methodology, utilizing a structured survey instrument to collect data from a representative sample of educators across various educational levels and cultural contexts. The survey is designed to measure key variables, including teachers' knowledge of ethnomathematical concepts, their attitudes toward culturally responsive teaching, and the challenges they face in implementing such practices. Statistical analysis will be used to identify trends, patterns, and correlations that shed light on the integration of cultural perspectives into mathematics education. The findings of this research aim to provide actionable insights for educators, curriculum developers, and policymakers. By highlighting the role of cultural knowledge in enhancing mathematical understanding, the study aspires to inform the creation of professional development programs and instructional resources that are both culturally responsive and pedagogically effective. This research will contribute to the growing field of ethnomathematics, fostering greater inclusivity and relevance in mathematics education worldwide.

Keywords

Ethnomathematics, in-service teachers, quantitative methodology, teacher beliefs, country comparison

CHARTING NEW FRONTIERS: CONFORTING THE CHALLENGES OF WEAVING THE HISTORY OF MATHEMATICS INTO UAE EDUCATIONAL FRAMEWORKS

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Abstract:

Charting New Frontiers: Confronting the Challenges of Weaving the History of Mathematics into UAE Educational Frameworks

This research represents the first comprehensive investigation into the challenges encountered by UAE K-12 teachers when attempting to integrate the history of mathematics (HoM) into their lessons. A total of 602 mathematics teachers from primary, middle, and high schools across all seven Emirates participated in this quantitative study. The research identifies several key obstacles, including time limitations, difficulties engaging students, gaps in teacher knowledge, and challenges in aligning historical content with the current curriculum. The study uses various methods to assess the scope and nature of these challenges, while also exploring their correlation with factors such as teaching experience, grade levels, and the teachers' academic backgrounds. Findings indicate that while the difficulty level varies, these challenges are pervasive and unrelated to the teachers' years of experience or their academic majors. These conclusions provide a foundation for curriculum reform and teacher training programs, underscoring the need for strategies to effectively integrate historical content within the framework of mathematics education, while adhering to national standards. The study's implications extend beyond the UAE, contributing to a broader global discussion on enhancing mathematics education through the inclusion of historical perspectives.

Keywords: History of mathematics, mathematics teaching in UAE, teachers' perceptions

2010 Mathematics Subject Classification: Mathematics Education

ADVANCING MATHEMATICAL PROFICIENCY AND TEACHER PREPARATION: INSIGHTS FROM THE MENTAL STARTERS ASSESSMENT PROJECT

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Abstract:

International benchmarking studies have highlighted the lack of mathematical proficiency among South African learners. Many researchers (e.g., Adler, 2009) suggest that teacher education institutions are complicit in the poor performance of learners in South Africa as pre-service teachers are not prepared to teach mathematics in such a way to develop flexibility, efficiency and accuracy when calculating. To address this challenge, academic researchers, in collaboration with the South African Department of Basic Education, developed the Mental Starters Assessment Project. This initiative focuses on equipping Grade 3 learners with mental strategies to work efficiently, accurately, and flexibly with numbers (Graven et al., 2020). While the Mental Starters Assessment Project was designed for in-service teachers, recently, the project has been introduced in pre-service teacher education programmes. The research asks the question: How does the implementation of the Mental Starters Assessment Project impact Grade 3 learners' mathematical proficiency and contribute to the professional development of pre-service teachers in the context of teaching mental mathematics strategies?

A key component of the project involved engaging Year 3 Bachelor of Education (Foundation Phase) pre-service teachers in teaching the bridging through ten strategy through a structured teaching sequence. The study examined two dimensions: (1) the impact of the intervention on 373 Grade 3 learners, assessed by the pre-service teachers through pre- and post-tests of the bridging through ten strategy, and (2) the professional learning of 39 pre-service teachers as they participated in workshops, implemented the teaching sequence during their Teaching Practice, analysed their classes results, and reflected on their experiences through questionnaires and focus group interviews. The research data for this paper was generated from pre- and post-tests across 12 Grade 3 classes. The pre-service teachers marked all the tests and entered them onto a spreadsheet that automatically calculated the results for each learner and the class average. To analyse the qualitative responses to the questionnaire and interviews, the participants' responses were coded according to the seven categories of Shuman's (1987) knowledge for teaching framework.

Findings indicate that the short-term, focused interventions significantly improved learners' mental mathematics proficiency. Simultaneously, pre-service teachers demonstrated growth in all seven of Shulman's knowledge for teaching categories. The study underscores the dual impact of the Mental Starters Assessment Project in enhancing learners' mathematical abilities while fostering critical pedagogical skills among pre-service teachers

Keywords: mathematical proficiency, knowledge for teaching, bridigng through ten strategy

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NINTH-GRADE STUDENTS' DİFFİCULTIES İN SOLVING SYSTEMS OF EQUATIONS WİTH THREE VARIABLES IN ABU DHABI HIGH SCHOOLS

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Abstract:

This study explores grade nine students' views about solving systems of equations with three variables in Abu Dhabi high schools. To figure out the challenges and difficulties that students face, the researcher conducted clinical interviews with five grade nine students with different performance levels. Each student was interviewed individually and asked them questions about different difficulty levels, starting with easy questions and ending with difficult questions. After the interviews, the results were presented using four themes: students' main struggles, students' emotions, students' choice of approach, and students' reflections. The revealed that the students have difficulties solving linear equation systems with three variables related to basic arithmetic skills, algebraic skills, and determining the best approach to solve the system of equations. Also, students felt anxious and unconfident when solving a system of equations with three variables. The study results will help mathematics teachers develop instruction methods to support students learning. The study rocommended that mathematics teachers should use some strategies to improve their students algebraic skills in solving systems of linear equations.

Keywords: Ninth-grade, difficulties, systems of linear equations

2010 Mathematics Subject Classification: Mathematics Education

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SS5 - Modeling and Simulation of Biological Systems

NEW OPTIMAL CONTROL APPROACH TO CONTAIN EPIDEMIC IN THE CASE OF LIMITED RESOURCES

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Abstract:

One of the significant challenges of public health authorities is controlling the spread of infectious diseases in a population. This problem can be even more challenging with limited resources, such as drug supplies, vaccination dosage, limited hospital beds, and healthcare professional workers (nurses and doctors). The world has faced this problem in the recent pandemic and other outbreaks, mainly in countries with economic difficulties and humanitarian crises. Mathematical modeling epidemic has proven to be a very effective tool for evaluating the impact of the control measures during outbreaks. These models are mainly systems of differential equations. The presence of limited resources requires the consideration of hybrid models that mimic the discontinuity on time of some variables or parameters in these models. Hence, using the traditional optimal control approach cannot be helpful. In this work, I will present a series of results from developing an optimal control of the switched epidemic model. The new approach relies on converting the switched system, a nonlinear, nonconvex optimal control problem, into an equivalent optimal control problem with a convex structure. This approach has proven effective in high-performance numerical computing.

Keywords: Switched System, Epidemic Model, Optimal Control

34A38, 92B05,49K15:

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Detection of Parkinson's disease through EEG brain signals using Open- Look, Listen and Learn (OpenL3) neural network & Yet Another Mobile Network (YAMNET) neural networks

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Abstract:

Parkinson's disease is a chronic neurodegenerative disease that affects the human central nervous system, with a variety of motor and non-motor symptoms. It is spreading rapidly, affecting millions of people worldwide [1]. The number of people with Parkinson's disease has increased from 1,617,000 in 2005 to 1,762,000 in 2015 and is expected to grow to 2,015,000 by 2030, according to recent statistics, indicating that the burden of Parkinson's disease is increasing [2].

Early detection of Parkinson's disease is important for helping to reduce symptoms and improve the quality of life for the patient in the future [3].

Recent scientific research has explored different advanced techniques for early detection of Parkinson's disease, including magnetic resonance imaging, electroencephalography, voice analysis, motion tracking, handwriting examination, etc., as these are the most widespread methods for early detection of the disease [4]. Among these techniques, as evidenced by current research, electroencephalography signals analysis is very effective in the early diagnosis of Parkinson's disease. They can be easily obtained without expensive equipment, allowing this technique to be extended to developing countries [5].

The complexity of EEG data requires the use of deep learning techniques to analyse it, a pioneering technique capable of handling complex signal patterns beyond the analytical capacity of human medical professionals [6]. The problem at hand is to classify the EEG to know if a person has Parkinson's disease or not using two sets of databases, the first for people with the disease and the second for healthy people.

In this study, unique applications have been used to detect Parkinson disease. Two deep neural networks called open-Look, Listen, and Learn (OpenL3) [7] and Yet Another Mobile Network (YAMNet) have been considered, that were pre-trained to classify audio signals. The EEG signals have been processed to obtain spectrograms, images representing the time variation of the signal power spectral density, which are used to feed the deep neural networks, in the same way of audio signals for audio classification. Before that, the signals are pre-processed with different types of filters [8].

The primary goal was to evaluate the effectiveness of the OpenL3, and YAMNet neural networks in distinguishing and classifying EEG signals across these groups. The results were obtained by feeding the deep learning model with the extracted features and using the transfer learning technique to reduce the training time. This study differs from others of a similar nature because it added noise to the original signal at various levels in order to train the model and get the best outcomes.

On the other hand, the OpenL3 method was found to show superior accuracy in the early detection of Parkinson's disease in the regression algorithm. These results suggest that the networks used in this study can serve as a valuable tool for detecting Parkinson's disease. Finally, this innovative approach holds great potential to enhance both early diagnosis and treatment of this condition.

Keyword: Electroencephalogram (EEG), deep learning, Parkinson disease, OpenL3, YAMNet.

2010 Mathematics Subject Classification:
68T - Artificial Intelligence and Machine Learning
68T05 - Artificial Neural Networks and Deep Learning
68T07 Artificial neural networks and deep learning
94A Communication, information
94A12 Signal theory (characterization, reconstruction, filtering, etc.)

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EXISTENCE CHAOS AND SHRIMPS LIKE PERIODIC ISLANDS IN ISO-SPIKES BIPARAMETRIC DIAGRAMS OF A TRI-TROPHIC FOOD WEB MODEL

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Abstract:

Complexity in food web systems are ubiquitously present in natural ecosystems due to environment variations, dynamically varying parameter and complex biotic and abiotic interactions. The occurrence of chaos in the dynamics of food web systems are recognized by several existing ways of nonlinear theory. Iso- spikes (dense with shrimp-like periodic-islands) is the one among them to identify different periodic islands in parameters space. In this study, a tri-trophic food chain model having prey protection against predation, provision of an additional food to top predator, and intra-species competition among the predators is explored locally and globally. The parametric conditions for species coexistence and diversity are established analytically. Further, numerical simulations verify these conditions through one-parameter and two-parameter bifurcation diagrams. One parameter bifurcation analysis reveals that only prey refuge or additional food can't be able to control the chaos of the food chain model but their combined effects with intra-species competition leads to a point stability. The two-parameter bifurcations provides a detailed analysis of how prey refuge, additional food, and intra-species competition collectively shape the system's behaviour. The isospike which contains periodic islands is analysed numerically to understand the route of chaos in the systems and the impact of refuge as well as additional food. Our findings suggest that intra-species competition, along with prey refuge and additional food for the top predator, play a crucial role to control chaos and bring the system into order. These dynamics ultimately contribute to higher species coexistence and increased biodiversity.

Keywords: Chaos, Tri-trophic Food web Systems, Periodic Islands, Species Coexistence

2010 Mathematics Subject Classification:

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MODELLING THE FEAR FACTOR AS DELAY SPATIOTEMPORAL EPIDEMIC MODEL

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Abstract:

This paper examines a Susceptible-Protected-Infected-Recovered (SPIR) epidemic model. This model incorporates a composite diffusion framework which encompasses both local and nonlocal diffusions focusing on the analysis of the influence of fear of infectious transmissions within the population. Our main goal in this investigation is to establish a mathematical model which is well-posed i.e. entails the validation of the existence, positivity, and uniqueness of its solution. A pivotal aspect of our analysis revolves around deriving a variational expression for the basic reproduction number ($\mathfrak{R}_0$). This parameter takes on the fundamental role of a threshold quantity for the dynamics of the epidemic. Specifically, when $\mathfrak{R}_0$ falls below 1, we substantiate that the epidemic will eventually diminish which indicates the global asymptotic stability of the disease-free equilibrium state. Conversely, when $\mathfrak{R}_0$ exceeds 1, we explain that the solution will persist uniformly, and endemic equilibrium state will exist. The global stability of this endemic equilibrium is demonstrated by employing Lyapunov function techniques. We consider two distinct cases: in the first case we consider that the diffusion coefficient for susceptible population is null i.e. zero, and second one corresponds to the case when the diffusion coefficient for infected population is null. In addition, we made a comparative analysis with the classical SIR epidemic model to elucidate the required protective measures necessary for disease control. And this is attempted through the reduction of $\mathfrak{R}_0$ below unity. The success of our goal depends on the implementation of suitabble protective measures.

Keywords: Fear; Global stability; Non-local diffusion.

2010 Mathematics Subject Classification: Modeling and Simulation of Biological Systems

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STABILITY OF THRESHOLD BOOLEAN NETWORKS

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General area of research: Modeling and Simulation of Biological Systems

Abstract:

For a cell to function properly, it must be able to rapidly adjust its gene regulation in response to changes in its environment, such as shifts in nutrient availability or temperature. Understanding how cells regulate gene expression in response to these external stimuli provides valuable insights into their adaptive mechanisms and overall survival strategies. Gene Regulatory Networks (GRNs) are usually studied mathematically as dynamical system models where the expression of a gene is assumed to be changing over time.

A Boolean network with n variables consists of a function $F = (f_1, f_2, \ldots, f_n) : \{0,1\}^n \to \{0,1\}^n$ where each Boolean function $f_i : \{0,1\}^n \to \{0,1\}$ determines the value of the variable x_i at time $t+1$, based on the input vector $X = (x_1, x_2, \ldots, x_n)$ at the previous time-step t. Despite their simplicity, these networks prove to be effective models for GRNs by representing the states of genes as either "on" or "off," depending on their expression levels [1]. To model GRNs as Boolean networks, directed graphs are used where the vertices represent genes, the edges represent interactions between them, and each gene is associated with a Boolean function that determines its state in response to changes in the states of its regulators.

Threshold Boolean Networks (TBNs) are a specialized type of Boolean network that function as discrete dynamical systems, where the state of each variable is determined by threshold-based rules. These networks have been extensively studied for their stability characteristics, particularly in fields such as physics and computer science, including applications like spin glasses and artificial neural networks. Recently, threshold Boolean functions have been applied in GRN models, revealing properties that closely resemble those observed in real biological cells [2, 3].

In this work, we explore the properties of the dynamics of TBNs and their relevance to gene regulatory models. Furthermore, we propose a new approach to assess the robustness of these networks, addressing the issue of multiple attractors [4]. This method suggests the existence of a set of dominant attractors in the dynamics of TBNs, which is not very common in Kauffman networks. Additionally, we conduct a comparative analysis between the dynamics of TBNs and Random Boolean Networks (RBNs), focusing on variations in the number of inputs per variable. These experiments include the number and length of attractors in both types and the distribution of states across attractors. Finally, we conduct a sensitivity analysis to explore the stability of the dominant attractors in TBNs.

Keywords: Boolean Networks, Threshold functions, Attractors.

2010 Mathematics Subject Classification: 37N25

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DYNAMICS OF HIV-1 AND CD4$^+$ T CELLS INTERACTIONS VIA PSEUDOSPECTRUM

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General area of research: Applied Mathematics

Abstract: A mathematical model of the process of interaction between HIV-1 (human immunodeficiency retrovirus) and CD4$^+$ T helper cells was investigated in the paper [1]. The model is based on 4 variables: uninfected CD4$^+$ T cells, infected CD4$^+$ T cells, infectious HIV virus, and the non-infectious free virus, and it has two equilibriums: disease-free equilibrium and seropositivity equilibrium. Under the condition $R_0 < 1$, where R_0 the basic productive number, local and consequently global stability is proved for both disease-free and seropositivity equilibrium. Local stability is based on the Lyapunov condition, which requires that the spectrum of the Jacobian at the equilibrium point is entirely in the open left complex half-plane. Under the same condition $R_0 < 1$, global stability is also guaranteed. However, the proven local (even global) stability is only *asymptotic*. Namely, the spectrum in the left complex half-plane tells us that, after the equilibrium has been disturbed, the system will *sometime* return to it, but it is impossible to see *when* that will be. Also, it is important to look at what happens to the system in the meantime, because perhaps such a transitional amplification occurs, that it violates the internal integrity of the system, before the asymptotic is manifested. Therefore, it is necessary to discuss the *robust* stability, which should be understood as the stability of a dynamical system together with ε -perturbed dynamical systems, for a sufficiently small ε given in advance. A mathematical tool that helps in this is pseudospectrum theory, see [2, 3]. In reality, dynamical systems are subject to significant stochastic variations. Incorporation of this stochastic behavior can make the system behave quite differently from its deterministic counterpart. Therefore, it is important to analyze how the above stability behaves in the presence of stochastic fluctuations. In other words, it is important to answer the following question: *To which extent the uncertainties in the parameters that define the deterministic model preserve the local asymptotic stability?* In addition, the use of the pseudospectrum solves the problem of *empirical noise*, i.e. measurement inaccuracies, as well as model inaccuracies in general. Motivated by these observations, we will conduct the dynamic analysis of the model proposed in [1] using the pseudospectrum. How much new information about the model can be obtained in this way is illustrated by the same examples given in [1].

Keywords: dynamical systems, stability, robust stability, pseudospectrum.

2010 Mathematics Subject Classification: 15A18, 65F99

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NUMERICAL SIMULATION OF LUNG TISSUE FREEZING DURING CRYOSURGERY USING TPL BIOHEAT MODEL

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Abstract: Cryosurgery or cryotherapy is a type of surgery in which the abnormal tissue is destroyed by generating an extremely low temperature in the targeted tumor tissue using probes. A successful treatment requires a cryoprobe with high cooling efficiency for destroying target tumor tissues. Various mathematical models have been developed for the heat transfer in biologial tissue. These can be characterized as parabolic, single phase lag, and dual phase lag models.

Although the single-phase lag bioheat model considers the microscale response in time [1], it does not consider the microscale response in space. So, Tzou [1] suggested a new model to improve the deficiency of microstructural effect in the fast transient process of heat transfer, considering the phase lag in heat flux (τ_q) and temperature gradient (τ_t). This model is known as the dual-phase lag (DPL) model. Recently, Choudhuri [2] constituted a three-phase-lag (TPL) constitutive model, which is able to describe the previously developed parabolic and hyperbolic model. The three phase lag model provides a theoretical and preferable heat conduction model that considers the medium's microstructure to accurately describe its temperature distribution.

The present study is concerned with the numerical study of a two-dimensional phase change problem in biological tissues during cryosurgery. We use the three-phase lag (TPL) bioheat model to examine the location of the freezing interface and the temperature distribution throughout the tissue. We apply the effective heat capacity formulation to solve the nonlinear governing equation. To approximate the time and space derivatives, the Crank-Nicolson (C-N) finite difference approximation and the Gaussian radial basis function, respectively, are employed. We study the impact of phase lag (τ_v) on phase change interface position and thermal distribution inside the tissue.

Keywords: Bioheat transfer, Three phase lag, Cryosurgery

2010 Mathematics Subject Classification: 62P10, 35Q92, 65D25, 65M70, 65M06

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Agent-Based Modeling to Enhance In Vitro Studies of Tuberculous Granuloma Formation

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General area of research: Modeling and Simulation of Biological System

Abstract: Tuberculosis (TB), caused by Mycobacterium tuberculosis (Mtb), is a global health challenge. Understanding the dynamics of granulomas, immune cell agglomerates central to TB pathology, progression and treatment, is crucial for anti-TB drug development. In vivo models face ethical and practical limitations, while in vitro Granuloma-Like Structures (GLSs), that include immune components, offer controlled environments to study granulomas and test drugs and host-directed therapies [1]. However, GLS experiments are slow, complex, and biologically variable. To address these limitations, we developed a customized agent-based model (ABM) inspired by the ERA4TB consortium's goal (www.era4tb.org) to advance TB therapies.

In vitro GLS experiments use blood cells from healthy donors, then infected with Mtb, and monitored for 10-15 days to assess bacterial viability via colony counting over time (CFU) and GLS size [4, 1, 2]. Using an adapted PhysiCell framework [3], our ABM simulates GLS formation with macrophages, T cells, chemokines, and cytokines. Pro- and anti-inflammatory cytokines mediate immune responses, driving macrophage adhesion, reduced motility, and differentiation. The model covers a portion (e.g. 1/10) of an experimental well, with cell and cytokine dynamics on minute- and second-scale time steps.

Our ABM capture GLS dynamics, aligning with experimental data. Variability in cellular behaviors highlights the influence of host traits on Mtb response, supporting therapies targeting the immune system.

We use Approximate Bayesian Computation (ABC) to calibrate the ABM [5], offering insights into parameter distributions and interactions. Despite its high computational cost and challenges in defining objectives, ABC could serve as an effective approach for validating agent-based models and advancing healthcare applications by enabling robust parameter inference. Future work will incorporate drug effects and simulate immune failure.

Keywords: Agent-Based Models, Simulations, Tuberculosis

2010 Mathematics Subject Classification: 92-08, Computational methods for problems pertaining to biology

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LEVERAGING DIFFERENTIAL EXPRESSION NETWORK ANALYSIS TO ADDRESS KEY QUESTIONS IN CANCER IMMUNOTHERAPY

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Abstract:
Cancer immunotherapy, which employs antibodies to target immune checkpoints, has demonstrated remarkable clinical success. However, only a subset of patients exhibits a positive response, and the biological mechanisms underlying treatment efficacy remain poorly understood. This knowledge gap hinders the development of effective combination therapies. To investigate this, mouse clinical experiments were conducted to compare cellular composition and gene expression profiles between responsive and nonresponsive tumors before Immune Checkpoint Blockade therapy. A computational mathematical approach utilizing Quadrant Scan is proposed to transform gene expression profiles and construct a differential expression network. This approach provides deeper insights into gene dynamics throughout treatment and helps predict patient response, guiding the need for additional therapeutic interventions to advance the response.

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Keywords: Cancer immunotherapy, Quadrant Scan, Differential Expression Network.

2010 Mathematics Subject Classification: Computational Mathematics, Mathematical Biology

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SS6 - Operator Theory and Harmonic Analysis

IMPROVED HARDY INEQUALITIES

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Abstract:

We discuss novel improvements of the classical discrete and continuous Hardy inequalities, which give the discrete and continuous versions of a recent (continuous, 1D) inequality of Frank, Laptev, and Weidl. Our arguments build on certain weighted inequalities based on various analogues of symmetric decreasing rearrangement techniques. This talk is based on our joint works [1], [2] and [3].

Keywords: Hardy inequality, Rellich inequality, sharp constant.

2010 Mathematics Subject Classification: 26D10; 35A23; 46E35

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[1] A. Orazbayev, D. Suragan, Improved Lp-Lq Hardy inequalities. Mathematical Inequalities and Applications, 27 (2024), no. 4.

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Hausdorff Operators on an Interval: Boundedness in Weighted Lebesgue Spaces

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Abstract: We study integral operators on the interval of real line which naturally arise in some problems in the theory of integral equations and mathematical physics. We study boundedness in weighted Lebesgue spaces: sufficient and necessary conditions for boundedness are given. Also, special important particular cases of operators and spaces are considered as examples. We construct identity approximation within the framework of the class of operators under study, and two different approaches are given.

Keywords: Hausdorff operators, Operators with homogeneous kernels, Weighted Lebesgue spaces.

2010 Mathematics Subject Classification: 45P05, 47B38, 26A06, 44A35, 45E10

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ON QUADRATURE-DIFFERENCE METHODS FOR SOLVING PERIODICS LINEAR AND NONLINEAR SINGULAR INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract: For the linear

$$\sum_{\nu=0}^{m}(a_\nu(t)x^\nu(t)+b_\nu(t)(Jx^\nu)(t)+(J^0h_\nu x^\nu)(t))=y(t)$$

and nonlinear

$$F(t,x^m,\dots,x(t),Jx^m)(t),\dots,(Jx)(t),(J^0h_mx^{(m)}(t),\dots,(J^0h_0x)(t))=y(t)$$

singular integro-differential equations where $x(t)$ is the dtsiered unknown, a_ν, $b_\nu(t)$, h_ν, $\nu=0,1,\dots,m$, $y(t)$ and $F(t,u_m,\dots,u_0,v_m,\dots,v_0,w_m,\dots,w_0)$ are given continuos functions, 2π-periodic in the variables t, τ, singular integrals

$$(Jx^\nu)(t)=\frac{1}{2\pi}\int_0^{2\pi}x^\nu(\tau)\cot\frac{\tau-t}{2}d\tau,\qquad \nu=0,1,\dots,m,$$

are to be interpreted as the Cauchy-Lesbegues principal values and

$$(J^0h_\nu x^{(\nu)})(t)=\frac{1}{2\pi}\int_0^{2\pi}h_\nu(t,\tau)x^\nu(\tau)d\tau,\qquad \nu=0,1,\dots,m,$$

are regular integrals the quadrature-differences methods are constructed and justified. The convergence of the methods is proved and the error estimations are obtained.

Multidimensional case is also considered.

Keywords: Singular integro-differential equations, quadrature-difference method, cubature-difference method

2010 Mathematics Subject Classification: 45E05 45L05 65R20

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MULTILINEAR SOBOLEV INEQUALITIES FOR FRACTIONAL INTEGRALS DEFINED BY MEASURE, AND RELATED TOPICS

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Abstract: Necessary and sufficient conditions on a measure μ guaranteeing the boundedness of the multilinear fractional integral operator $T^{(m)}_{\gamma,\mu}$ (defined with respect to a measure μ) from the product of Lorentz spaces $\prod_{k=1}^{m} L^{r_k,s_k}_{\mu}$ to the Lorentz space $L^{p,q}_{\mu}(X)$ are derived. The results are new even for linear fractional integrals $T_{\gamma,\mu}$ (i.e., for $m = 1$). From the general results we obtain a criterion for the validity of the Sobolev inequality for $T^{(m)}_{\gamma,\mu}$ in Lorentz spaces defined with respect to μ. We investigate the same problem for Morrey-Lorentz spaces. Similar problem for the classical Lebesgue spaces were derived in [1] (see [2] for the linear case).

Criteria for the boundedness of $m-$ linear Riemann-Liouville operators will also be discussed.

Some of these results were announced in [3], [4].

The investigation was carried out jointly with L. Natelashvili.

Keywords: Fractional integrals, Sobolev Inequality, Lorentz spaces.

2010 Mathematics Subject Classification: 26A33; 42B35; 46B70; 47B38.

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ON THE WELL-POSEDNESS OF THE LOCAL BOUNDARY VALUE PROBLEM FOR THE INVOLUTORY ELLIPTIC EQUATION

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Abstract: In the present paper, the boundary value problem for the involutory elliptic equation in a Banach space with a positive operator is investigated. The main theorem on well-posedness of this problem is established. In practice, the coercive stability estimates for solution of several types of boundary value problems for elliptic differential equations are proved.

Keywords: Involutory elliptic equation,well-posedness, positive operator.

2010 Mathematics Subject Classification:35L10, 35L90, 35B35

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ALGEBRAIC PROPERTIES OF DIMOVSKI SPACES FOR OPERATIONAL CALCULUS

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Abstract:

The classical operational calculus of Mikusiński is performed in the function space $C[0,\infty)$ of continuous functions on the closed half-line. Algebraic properties of this space under addition and convolution [3] can be used to solve ordinary differential equations, by a method formally similar to that of Laplace transforms but applicable on larger function spaces [2].

Several variants of this theory have been used to solve differential equations involving other operators, particularly those of fractional order [4]. One of the most important function spaces for such variants is the following extension of $C[0,\infty)$ due to Dimovski:

$$C_\alpha[0,\infty) = \Big\{ f(x) = x^p f_1(x) \ : \ f_1 \in C[0,\infty),\, p > \alpha \Big\},$$

where $\alpha \in [-1,\infty)$ is a fixed real number, often taken to be $\alpha = -1$ for a space that nestles between $L^1_{loc}[0,\infty)$ and $C[0,\infty)$. We investigate algebraic properties of this space, including the structure of its ideals and its Jacobson radical. The latter leads to a general result on the invertibility of any linear operator given by convolution with an element of C_α combined with the identity. Our results have been published in [1].

Keywords: Mikusiński's operational calculus; ideal theory of rings; inversion of functional operators.

2010 Mathematics Subject Classification: 44A40; 13A15; 26A33

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IMPROVED HARDY INEQUALITIES

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Abstract:

We discuss novel improvements of the classical discrete and continuous Hardy inequalities, which give the discrete and continuous versions of a recent (continuous, 1D) inequality of Frank, Laptev, and Weidl. Our arguments build on certain weighted inequalities based on various analogues of symmetric decreasing rearrangement techniques. This talk is based on our joint works [1], [2] and [3].

Keywords: Hardy inequality, Rellich inequality, sharp constant.

2010 Mathematics Subject Classification: 26D10; 35A23; 46E35

References

[1] A. Orazbayev, D. Suragan, Improved Lp-Lq Hardy inequalities. Mathematical Inequalities and Applications, 27 (2024), no. 4.

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A PARTIAL SOLUTION TO RICCERI'S CONJECTURE

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Abstract: In [7] Ricceri establishes the notion of total anti-proximinality and poses a conjecture on the topological structure of such sets.

- Let E be a metric space. A non-empty proper subset A of E is called anti-proximinal when for every element $e \in E \setminus A$ the distance from e to A, $d(e, A)$, is never attained at any $a \in A$.
- Let E be a vector space. A non-empty proper subset A of E is called totally anti-proximinal when A is anti-proximinal for every norm on E.
- A Hausdorff locally convex topological vector space E is said to have the anti-proximinal property whenever every totally anti-proximinal convex subset is not rare.
- A Hausdorff locally convex topological vector space E is said to have the weak anti-proximinal property whenever every totally anti-proximinal absolutely convex subset is not rare.

Concerning the existence of bounded convex anti-proximinal sets in Banach spaces see [1, 2]. Recall that a subset of a topological space is said to be rare when its closure has empty interior. Ricceri's Conjecture [7] reads as follows: *There exists a non-complete normed space enjoying the anti-proximinal property.* We will review the most recent results about this conjecture [3, 4, 5, 6] and some other future results.

Keywords: Topological vector space; Convex set; Balanced set; Non-rareness; Anti-proximinality.

2010 Mathematics Subject Classification: 46A03, 46A55

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Almost everywhere convergence of subsequences of partial sums of Fourier series with respect to Vilenkin systems

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The classical theory of Fourier series deals with decomposition of a function into sinusoidal waves. Unlike these continuous waves the Vilenkin (Walsh) functions are rectangular waves (for details see the book [1]). There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, combined with martingale theory.

In [2], using a novel technique in martingale theory, a new proof was proposed for an analogy of the famous Carleson-Hunt theorem for Fourier series with respect to the Vilenkin system for any $f \in L_p$, for any $p > 1$. Moreover, it was also proved the analogy of Kolmogorov theorem and was found a new construction of an integrable function $f \in L_1$ such that partial sums of Vilenkin-Fourier series diverges everywhere.

This talk is devoted to characterize some subsequences of natural numbers such that partial sums of Vilenkin-Fourier series with such indices of any integrable function $f \in L_1$ converges almost everywhere to this function (for details see [3]).

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ON THE COMMUTING PROBLEM OF TOEPLITZ OPERATORS ON THE HARMONIC BERGMAN SPACE

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General area of research: Analysis

Abstract: Many algebraic properties of Toeplitz operators have been studied extensively in the literature, one of the most interesting algebraic properties is the commuting problem. Recall that two operators T_1 and T_2 are said to commute iff their commutator $[T_1, T_2] = T_1T_2 - T_2T_1 = 0$, i.e, $T_1T_2 = T_2T_1$.
In this talk, we present our contributions toward solving this problem. More precisely we show that that if a Toeplitz operator T_f on the harmonic Bergman space, with a truncated above symbol f commutes with a Toeplitz operator with the symbol $z + \bar{g}$, then T_f will be a nontrivial linear combination of $T_{z+\bar{g}}$ and the identity operator I.This work is a joint work with A.Yousef and I.Louhichi

Keywords: Toeplitz operators, harmonic Bergman space, Bergman space

2010 Mathematics Subject Classification: Primary 47B35; Secondary 47B38.

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SLOW PROPAGATION VELOCITIES IN SCHRÖDINGER OPERATORS WITH LARGE PERIODIC POTENTIAL

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General area of research: Operator Theory and Harmonic Analysis.

Abstract:

Schrödinger operators with periodic potential have generally been shown to exhibit ballistic transport. In this work, we investigate if the propagation velocity, while positive, can be made arbitrarily small by a suitable choice of the periodic potential. We consider the discrete one-dimensional Schrödinger operator $\Delta + \mu V$, where Δ is the discrete Laplacian, V is a p-periodic non-degenerate potential, and $\mu > 0$. We establish a Lieb-Robinson-type bound with a group velocity that scales like $\mathcal{O}(1/\mu)$ as $\mu \to \infty$. This shows the existence of a linear light cone with a maximum velocity of quantum propagation that is decaying at a rate proportional to $1/\mu$. Furthermore, we prove that the asymptotic velocity, or the average velocity of the time-evolved state, exhibits a decay proportional to $\mathcal{O}(1/\mu^{p-1})$ as $\mu \to \infty$.

This presentation is based on the publication [1].

Keywords: Periodic Schrödinger operators, ballistic transport, propagation velocity

2010 Mathematics Subject Classification: 47B36, 47N50, 81Q10, 82B44.

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CARLESON-TYPE EMBEDDINGS WITH CLOSED RANGE

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Abstract: We characterize the Carleson measures μ on the open unit disk, $\mathbb{D}$, such that the image of the Hardy space H^p under the corresponding embedding operator is closed in $L^p(\mathbb{D}, \mu)$. In fact, a more general result involving (p, q)-Carleson measures is obtained. A similar problem is solved in the context of Bergman spaces.

Keywords: Carleson measure, Hardy space, Bergman space

2010 Mathematics Subject Classification: 30H10, 46E15

Convolutions on Lie groups, Generic Bessel Potential Spaces and applications

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The purpose of the presentation is to discuss the convolution integro-differential equations on Lie groups and their applications to some equations of Mathematical Physics. In this framework we suggest to underline the role of Generic Bessel potential spaces (GBPS) to the structure of underlying Lie group. Definition of GBPS are based on generic differential operators from the Lie algebra of the Lie group. Such generic Bessel potential spaces are adapted better to the investigation of integro-differential (of pseudo-differential) operators on Lie groups.

We concentrate investigation on a Lie groups $\{G, x \circ y\}$ with the group operation $x \circ y$, which are homeomorphic to the Lie group $\{\mathbb{R}^n, x \circ y = x + y\}$. Then on $\{G, x \circ y\}$ we have uniquelly defined Haar measure $d_G\mu$, the Fourier transform $\mathcal{F}_G$, its inverse $\mathcal{F}_G^{-1}$ and generic differential operators $\mathfrak{D}_1, \dots, \mathfrak{D}_n$, generated by the vector fields from the corresponding Lie algebra. The dual group is then $\widehat{G} = \mathbb{R}^n$ and Convolution operators are

$$\boldsymbol{W}^0_{a,G} := \mathcal{F}_G^{-1} a \mathcal{F}_G \ : \ \mathbb{S}(G) \to \mathbb{S}'(G), \tag{1}$$

where the symbol $a(\xi)$ is a distribution on the dual group $a \in \mathbb{S}'(\widehat{G}) = \mathbb{S}'(\widehat{\mathbb{R}^n})$, $\mathbb{S}(G)$ is the Schwartz spaces of fast decaying smooth functions and $\mathbb{S}'(G)$ is the spaces of distributions.

We will expose several examples of Lie groups and corresponding GBPS. Then we concentrate on the investigation of boundary value problems (BVPs) for the Laplace-Beltrami equation on a hypersurface $\mathcal{C}$ with the Lipschitz boundary, containing a finite number of angular points (knots).

The first part of the presentation is based on joint research with M. Ruzhanski, D, Cardona, A. Hendrix (Ghent) and the second part-on joint research with M. Caava (Tbilisi).

A FRACTIONAL LAPLACIAN AND ITS EXTENSION PROBLEM

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Abstract:

We will present four equivalent characterizations of the fractional Dunkl-type Laplacian, an operator that combines both differential and difference terms. We begin with the natural Fourier characterization of the fractional Laplacian, and then we will prove both Bochner's characterization and the singular integral formulation of this operator. A significant portion of the presentation will focus on the fourth characterization, where we derive the fractional Laplacian as a Dirichlet-to-Neumann map through an extension problem in the upper half-plane. Additionally, we will obtain a Poisson formula for the extension.

Keywords: Fractional Laplacian, Heat semigroup, Extension problem, Poisson kernel.

2010 Mathematics Subject Classification: 42B25

STRONG AND WEAK ASSOCIATED REFLEXIVITY OF CERTAIN FUNCTION CLASSES

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Abstract: The report provides an overview of recent results on the problem of describing associated and doubly associated spaces to functional classes that include both ideal and non-ideal structures. The latter include first-order two-weight Sobolev spaces on the positive semiaxis [1]. It is shown that, unlike the concept of duality, associativity can be "strong" and "weak". At the same time, the doubly associated spaces are divided into three more types. In this context, it is established that the space of Sobolev functions with compact support has weakly associated reflexivity, and strongly associated with a weakly associated space consists only of zero [2]. Weighted spaces of Cesaro and Copson type have similar properties, for which the problem has been fully studied and their connection with Sobolev spaces with power weights [3] has been established. As an application, the problem of the boundedness of the Hilbert transformation from the Sobolev space to the Lebesgue space [4] is considered.

The study was performed at the Steklov Mathematical Institute and supported by the Russian Science Foundation (project 24-11-00170, https:// rscf.ru/project/24-11-00170/).

Keywords: Non–ideal function space, associated reflexivity, weighted Sobolev space.

2010 Mathematics Subject Classification: 46E35, 446E30.

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RECTANGULAR HARDY OPERATOR IN WEIGHTED LEBESGUE SPACES

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Abstract:

Let $n \in \mathbb{N}$. For Lebesgue measurable functions on $\mathbb{R}^n_+ := (0,\infty)^n$, the n–dimensional rectangular Hardy integral operator is given by the formula

$$I_n f(x_1,...,x_n) := \int_0^{x_1} ... \int_0^{x_n} f(y_1,...,y_n)\, dy_1...dy_n \qquad (x_1,...,x_n > 0). \tag{1}$$

Let $0 < p,q \le \infty$, $1 \le p \le \infty$ and $v, w \ge 0$ be weights on $\mathbb{R}^n_+$. Weighted Lebesgue space $L^p_v(\mathbb{R}^n_+)$ consists of all measurable on $\mathbb{R}^n_+$ functions f such that $\|f\|^p_{p,v} = \int_{\mathbb{R}^n_+} |f|^p v < \infty$.

The problem considered in this work is to characterise the integral inequality

$$\|I_n f\|_{q,w} \le C_n \|f\|_{p,v} \tag{2}$$

for all $f \in L^p_v(\mathbb{R}^n_+)$ and to describe the properties of the Hardy operator.

The characterisation of the inequality (2) is equivalent to the boundedness problem of operator (1) in weighted Lebesque spaces. By now, the case $n = 1$ has been completely studied (see, e.g. [3, 2]). Multidimensional inequalities of the form (2) were considered in e.g. [1] under certain restrictions on weights. The talk is mainly based on results from the papers [7, 6, 5, 4] devoted to the case $n = 2$.

Keywords: Hardy integral operator, Lebesgue space, boundedness.

2010 Mathematics Subject Classification: 26D10, 47G10.

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ATOMIC DECOMPOSITION FOR THE BOURGAIN-BREZIS-MIRONESCU SPACE

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Abstract: Given a Banach space E with a supremum-type norm induced by a collection of operators, we prove that E is a dual space and provide an atomic decomposition of its predual. We apply this result to the function space $\mathcal{B}$ introduced recently by Bourgain, Brezis, and Mironescu. This yields an atomic decomposition of its predual. This is a joint work with Luigi Greco, Karl-Mikael Perfekt, Carlo Sbordone and Roberta Schiattarella [1].

Keywords: Dual and predual, Atomic decomposition

2010 Mathematics Subject Classification: 46E99, 46E30, 46S99.

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[1] **Luigi D'Onofrio, Luigi Greco, Karl-Mikael Perfekt, Carlo Sbordone and Roberta Schiattarella**, Atomic decompositions, two stars theorems, and distances for the Bourgain–Brezis–Mironescu space and other big spaces. *Ann. Inst. H. Poinc. Anal. Non Lin.*, 2020, 37, no. 3, pp. 653–661

Frequent Hypercyclicity of some non-convolution operators

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Abstract: A hypercyclic operator is one that has a vector with a dense orbit. A frequently hypercyclic operator is an operator that has a hypercyclic vector such that the number of times that the orbit visits any non-empty open set, has positive lower density. While hypercyclicity criteria (and their frequent hypercyclic counterparts) provide sufficient conditions for hypercyclicity (frequent hypercyclicity), in this talk will showcase examples of (non-convolution) operators where the direct application of these criteria is not evident, necessitating the development of new ideas to establish frequent hypercyclicity.

Keywords: Hypercyclic operator, non-convolution operator, frecuently hypercyclic operator

2010 Mathematics Subject Classification:

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Commutants of Certain Classes of Toeplitz Operators

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Abstract: One of the most intriguing problems in the theory of Toeplitz operators on the analytic Bergman space of the unit disk is the characterization of their commutants–that is, the set of all Toeplitz operators that commute with a given one. In this talk, we will survey classical results on this problem and present some of the latest developments in the field.

Keywords: Toeplitz operator, quasihomogeneous symbol, Mellin transform.

2020 Mathematics Subject Classification: Primary 47B35, Secondary 47L80

SERIES EXPANSIONS OF SOLUTIONS OF PARABOLIC DIFFERENTIAL-DIFFERENCE EQUATIONS

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Abstract:

Consider the Cauchy problem with a bounded initial-value function u_0 for the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}(x,t) + \sum_{k=1}^{m} a_k \frac{\partial^2 u}{\partial x^2}(x+h_k,t),$$

where $a_1,\dots,a_m$ and $h_1,\dots,h_m$ are real constants and there exists a positive constant C such that the inequality $1+\sum\limits_{k=1}^{m} a_k \cos h_k\xi \geq C$ holds on the whole real axis.

The following assertion holds.

Theorem.

In the half-plane $\{x\in\mathbb{R}, t>0\}$, *the solution is equal to*

$$\sum_{j=0}^{\infty} t^j \sum_{|\alpha|=j} \frac{\prod\limits_{k=1}^{m} a_k^{\alpha_k}}{\prod\limits_{k=1}^{m} \alpha_k!} \frac{\partial^{2j} v}{\partial x^{2j}}\left(x+\sum_{k=1}^{m}\alpha_k h_k, t\right), \tag{1}$$

where $v(x,t)$ *is the bounded solution of the Cauchy problem for the heat equation with the initial-value function* u_0, *while* $\alpha=(\alpha_1,\dots,\alpha_n)$ *is an* n*-dimensional multiindex. Series* (1) *uniformly converges on each compact subset of the half-plane* $\{x\in\mathbb{R}, t>0\}$.

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This work of the second and third author was supported by the Ministry of Science and Higher Education of the Russian Federation under project 0707-2020-0034.

Keywords: Parabolic equations, differential-difference equations.

2010 Mathematics Subject Classification: 35K59

DIRECT AND INVERSE PROBLEMS FOR THE POISSON EQUATION WITH EQUALITY OF FLOWS ON A PART OF BOUNDARY

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Abstract:

In this report, we consider a stationary diffusion problem described by the Poisson equation. The problem is considered in a model domain, chosen as a half-disk. Classical Dirichlet boundary conditions are set on the arc of the circle. New non-local boundary conditions are set on the bottom base. The first condition means an equality of flows through opposite radii, and the second condition is the proportionality of distribution densities on these radii with a variable coefficient of proportionality. Uniqueness and existence of the classical solution to the problem are proved.

An inverse problem for the solution to the Poisson equation and its right-hand part depending only on an angular variable are considered. As an additional condition we use the boundary over-determination. Inverse problems to the Dirichlet and Neumann problems, and to problems with non-local conditions of the equality of flows through the opposite radii are considered. The well-posedness of the formulated inverse problems is proved.

The report is based on the author's previous publications [1-2].

Keywords: Poisson equation; non-local boundary conditions; inverse problem.

2010 Mathematics Subject Classification: 35J05; 35J25; 35R30

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SS7 - Rings, Monoids, Module Theory and Graphs from Rings

Schreier and ⋆-modules

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Abstract:

Throughout this talk all rings are commutative with identity and all modules are considered unitary. We study the concepts of Schreier modules and ⋆-modules. Some equivalent conditions for a module to be a pre-Schreier module are provided. We introduce the concept of quasi Schreier module and study some properties of this class of modules. Then we define the concept of ⋆-module and obtain the relationship between pre-Schreier modules and ⋆-modules.

Keywords: Schreier module; ⋆-module; quasi schreier module.

2010 Mathematics Subject Classification: 16N99; Secondary 06C05

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EQUATIONS AND CHARACTERIZATIONS FOR INTERMEDIATE RINGS

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Abstract:

We present various characterizations and equations regarding the cardinality and length of the set of intermediate rings in extensions of commutative rings that satisfy some finiteness conditions and need not necessarily be integral domains. We also provide numerical characterizations for rings that satisfy specific related conditions.

Keywords: intermediate rings in ring extensions; finite direct product; length of a ring extension.

2010 Mathematics Subject Classification: 13B02, 13B22, 13E15, 13E99, 13F05, 13G05, 13B30

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One absorbing factorization lattices

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General area of research: Algebra and Its Applications

Abstract:

Let $\mathcal{L}$ be a C-lattice and let $x \in \mathcal{L}$ be proper. We say that x is a 1-absorbing element (OA-element) of $\mathcal{L}$ if for all proper $a, b, c \in \mathcal{L}$ with $abc \leq x$, it follows that $ab \leq x$ or $c \leq x$. Note that every prime element of $\mathcal{L}$ is an OA-element and every OA-element of $\mathcal{L}$ is primary. Furthermore, $\mathcal{L}$ is said to be an OAFL (resp. COAFL, resp. POAFL) if every element (resp. every compact element, resp. every principal element) of $\mathcal{L}$ is a finite product of OA-elements of $\mathcal{L}$. In this talk, we present some characterizations and properties of OAFLs, COAFLs and POAFLs and discuss their relations.

Keywords: 1-absorbing element, C-lattice, principally generated lattice

2010 Mathematics Subject Classification: 06F10, 06F05, 13A15

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DIRECT PROJECTIVE MODULES AND THEIR GENERALIZATIONS

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(joint work with Sonal Gupta)

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Abstract:

The concept of direct projective module which is a generalization of quasi-projective module, was introduced by W. K. Nicholson [3] in 1976. A right R-module M is said to be direct projective if given a direct summand N of M with projection $p: M \to N$ and any epimorphism $f: M \to N$ there exist a $g \in End_R(M)$ such that $fog = p$. In this talk we discuss some more properties on direct projective modules in terms of SSP, SIP, Rickart, Endoregular modules [1,2]. Also, we discuss about finite direct projective modules and Pure direct projective modules which are generalizations of direct projective modules.

Keywords: Direct Projective modules, Finite direct projective modules, Pure direct projective modules.

2010 Mathematics Subject Classification: 16D40, 16E50, 16E60.

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SOME COMMUTATIVITY RESULTS IN NEARRINGS

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Abstract: The algebraic structure nearring is a non-linear counterpart of more familiar structure ring. We present results which show that the existence of prime ideals, generalised derivations and certain conditions satisfied by commutators turn a nearring in to a commutative ring.

Keywords: Nearring, Commutativity, Derivation

2010 Mathematics Subject Classification: 16Y30

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THE n-TOTAL GRAPH OF A COMMUTATIVE RING

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Abstract:

Let $n \geq 1$. This thesis introduces the n-total graph of a commutative ring R. The n-total graph of a commutative ring R, denoted by $n - T(R)$, is an undirected simple graph with vertex set R, such that two vertices x, y in R are connected by an edge if $x^n + y^n$ in $Z(R)$. Note that if $n = 1$, then the 1-total graph of R is the total graph of R as in [1]. In this thesis, we study some graph properties, for example, connectedness, diameter, girth, for the graph that we defined.

Keywords: Commutative rings, Total graph, Zero-divisors.

2010 Mathematics Subject Classification: Primary 13A15; Secondary 13F99, 05C99.

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ON ϕ-$(2, J)$-IDEALS OF COMMUTATIVE RINGS

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General area of research: Ring Theory

Abstract: In this paper we introduce ϕ-$(2, J)$-ideals that is a generalization of $(2, J)$-ideals. Let R be a commutative ring with identity, $\phi : \mathcal{I}(R) \to \mathcal{I}(R) \cup \{\emptyset\}$ is a function where $\mathcal{I}(R)$ denotes the set of all ideals of R and I be a proper ideal of R. Then I is a called ϕ-$(2, J)$-ideal if whenever $a, b, c \in R$ and $abc \in I - \phi(I)$ then $ab \in I$ or $ac \in J(R)$ or $bc \in J(R)$. We give some properties of ϕ-$(2, J)$-ideals and investigate relations with other types of ideals such as $(2, J)$-ideals, weakly-$(2, J)$-ideals and ϕ-J-ideals. Moreover, we investigate the behaviour of ϕ-$(2, J)$-ideals under homomorphisms, in factor rings, in rings of fractions, in trivial extension, in cartesian product of rings.

Keywords: ϕ-$(2, J)$-ideal, $(2, J)$-ideal, J-ideal.

2010 Mathematics Subject Classification: 13A15, 13A18, 13A99.

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Polynomial Krull Domains

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Abstract: In this talk we present a classification of the class of Krull domains contained between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$, called Polynomial Krull domains. By a result of W. Heinzer ([3]), this class coincides with the class of Integrally Closed Noetherian domains between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$. The description is given by means of a generalization of the classical notion of rings of integer-valued polynomials, along the same way of the recent results about the classification of Polynomial Dedekind domains, that is, Dedekind domains between $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$ ([4, 5]). We accomplish this result by giving a full description of the class of Discrete Valuation Rings (DVRs) V with quotient field $\mathbb{Q}(X)$, which belong to two distinct classes, depending on whether $V \cap \mathbb{Q} = \mathbb{Q}$ or $V \cap \mathbb{Q} = \mathbb{Z}_{(p)}$, for some prime $p \in \mathbb{Z}$; the DVRs of the last case are distinguished into two subcases, according to whether the extension of the residue fields of V over $\mathbb{Z}_{(p)}$ is algebraic ([5]) or transcendental ([1]).

This is a work in progress, written jointly with G. W. Chang.

Keywords: Polynomial ring, Noetherian domain, Krull domain, integer-valued polynomials.

2020 Mathematics Subject Classification: 13B25, 13F05, 13F20

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ON DIFFERENT CLASSES OF PRIME HYPERIDEALS IN HYPERLATTICES

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Abstract:

The notion of hyperlattices is a generalization of the notion of lattice, wherein one (or both) of the binary operations is (are) replaced with hyperoperation (s). Wasadikar et. al. studied the generalizations of a prime ideal in lattices. In this presentation, we define and study various classes of prime hyperideals viz., 2-absorbing hyperideals, primary hyperideals, and their weak counterparts in meet hyperlattices. Further, we explore the properties of annihilators of these hyperideals and discuss their interrelations.

Keywords: Hyperlattices, hyperideals, 2-absorbing ideals.

2010 Mathematics Subject Classification: 06B75, 06B10

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HOMOLOGICAL CHARACTERIZATIONS OF G-KRULL DOMAINS AND G-DEDEKIND DOMAINS

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Abstract: Gorenstein Krull domains (G-Krull domains) are defined as domains R that satisfy the following conditions:

1. For each prime ideal $\mathfrak{p}$ of height one, $R_{\mathfrak{p}}$ is a Gorenstein ring.
2. $R = \bigcap R_{\mathfrak{p}}$, where $\mathfrak{p}$ ranges over all prime ideals of height one.
3. Any nonzero element of R lies in only a finite number of prime ideals of height one.

This study aims to characterize G-Krull domains from the perspective of Gorenstein homological algebra, analogous to the characterization of G-Dedekind domains. To achieve this, we introduce the concept of w-locally G-projective modules. An R-module M is called w-locally G-projective if $M_{\mathfrak{m}}$ is G-projective for every maximal w-ideal $\mathfrak{m}$ of R. We demonstrate that a domain R is G-Krull if and only if R is an SM domain and every w-ideal of R is w-locally G-projective. Additionally, we establish that a domain R is G-Dedekind if and only if R is a Noetherian domain and every maximal ideal of R is G-projective.

Keywords: G-Krull domain; SM domain; G-projective ideal

2010 Mathematics Subject Classification: 13C99, 13A15

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INVERSE SEMIGROUPS AND LEAVITT PATH ALGEBRAS

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Leavitt path algebras are linear associative algebras associated with directed graphs. They are closely related to graph C^*-algebras. Their development is an outgrowth of the work of W.G. Leavitt who showed in the 1960's that non-commutative rings fail to have the invariant basis number property (in a very strong sense).

With each directed graph we introduce an inverse semigroup that we refer to as the *Leavitt inverse semigroup* of the graph. The Leavitt inverse semigroup of a directed graph is a natural subsemigroup of the multiplicative semigroup of the corresponding Leavitt path algebra. We show that two directed graphs that have isomorphic Leavitt inverse semigroups have isomorphic Leavitt path algebras. By contracting spanning trees of certain subgraphs of a directed graph Γ to a point, we obtain a new directed graph $\bar{\Gamma}$ with the property that the Leavitt inverse semigroups of Γ and $\bar{\Gamma}$ are strongly Morita equivalent and the Leavitt path algebras of Γ and $\bar{\Gamma}$ are Morita equivalent. We make use of this construction to give necessary and sufficient conditions for two graphs to have isomorphic Leavitt inverse semigroups. As a consequence, we study some structural properties of Leavitt inverse semigroups and Leavitt path algebras, and we show in particular that Leavitt path algebras are 0-retracts of certain matrix algebras.

Key Words: Inverse semigroups, Leavitt path algebras.

2020 Mathematics Subject Classification: 20M18, 16S88

STUDY ON VARIANTS OF CHAIN CONDITION IN MODULES AND RINGS

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Abstract:

In this talk we would like to explore the study of ascending/ descending chain condition of variants of module and ring structures. Here we will try to explore several properties of Artinian/ Noetherian modules and rings and their generalizations in the various context of chain condition, also trying to establish the relationship of these modules with injective/ projective modules. Further, we will focus on well-known Hopkins-Levitzki theorem and Hilbert's Basis theorem.
We investigate whether Hopkins-Levitzki Theorem extend to semi-projective module. Unfortunately the answer we have is negative; counter example is provided. However it is shown that, the answer is positive for certain large classes of semi-projective modules.

Keywords: Ascending/ descending chain condition; e-Artinian modules/ rings; semi-projective module.

2020 Mathematics Subject Classification: 16D60, 16P20, 16P70

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CHARACTERIZATION OF GENERALIZED LIE TYPE DERIVATIONS ON RINGS AND ALGEBRAS

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Abstract: Let $\mathcal{R}$ be a commutative ring with identity, and $\mathcal{A}$ be an algebra over $\mathcal{R}$. An $\mathcal{R}$-linear mapping $\mathcal{L} : \mathcal{A} \to \mathcal{A}$ is called a *derivation* if $\mathcal{L}(XY) = \mathcal{L}(X)Y + X\mathcal{L}(Y)$ for all $X, Y \in \mathcal{A}$. Let $[X, Y] = XY - YX$ denote the Lie product of elements $X, Y \in \mathcal{A}$. An $\mathcal{R}$-linear mapping $\mathcal{L} : \mathcal{A} \to \mathcal{A}$ is said to be a *Lie derivation* if $\mathcal{L}([X, Y]) = [\mathcal{L}(X), Y] + [X, \mathcal{L}(Y)]$ for all $X, Y \in \mathcal{A}$.

Given the consideration of Lie derivations and Lie triple derivations, we can further extend them in a more general way. Suppose that $n \geq 2$ is a fixed positive integer. Let us consider the following sequence of polynomials: $p_1(X_1) = X_1, p_2(X_1, X_2) = [p_1(X_1), X_2] = [X_1, X_2], p_n(X_1, X_2, \cdots, X_n) = [p_{n-1}(X_1, X_2, \cdots, X_{n-1}), X_n]$ The polynomial $p_n(X_1, X_2, \cdots, X_n)$ is called a $(n-1)$-*commutator* $(n \geq 2)$. An $\mathcal{R}$-linear mapping $\mathcal{L} : \mathcal{A} \longrightarrow \mathcal{A}$ is called a *Lie n-derivation* if

$$\mathcal{L}(p_n(X_1, X_2, \cdots, X_n)) = \sum_{k=1}^{n} p_n(X_1, \cdots, X_{k-1}, \mathcal{L}(X_k), X_{k+1}, \cdots, X_n)$$

for all $X_1, X_2, \cdots, X_n \in \mathcal{A}$. Obviously, a Lie derivation is a Lie 2-derivation and a Lie triple derivation is a Lie 3-derivation. Lie 2-derivations, Lie 3-derivations and Lie n-derivations are collectively referred to as *Lie-type derivations*. Lie n-derivations have been further generalized as follows: Let $G_{\mathcal{L}} : \mathcal{A} \to \mathcal{A}$ be an $\mathcal{R}$-linear mapping and $\mathcal{L}$ be a Lie n-derivation on $\mathcal{A}$. Then $G_{\mathcal{L}}$ is called a generalized Lie n-derivation associated with the Lie n-derivation $\mathcal{L}$ if $G_{\mathcal{L}}(p_n(X_1, X_2, \ldots, X_n)) = p_n(G_{\mathcal{L}}(X_1), X_2, \ldots, X_n) + \sum_{k=2}^{n} p_n(X_1, \cdots, X_{k-1}, \mathcal{L}(X_k), X_{k+1}, \cdots, X_n)$ for all $X_1, X_2, \cdots, X_n \in \mathcal{A}$. *Lie-type derivations* have extensively been studies by many authors (see e.g. [1, 2, 3, 4] and references therein). In the present talk, the afore-mentioned developments will be discussed in details and some potential future research problems in this direction will also be provided.

Keywords: triangular algebra, Lie n-derivation, generalized Lie n-derivation.

2010 Mathematics Subject Classification:16N60, 16W25, 46L10

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PRIME-COEFFICIENT GRAPHS FROM THE GAUSSIAN RING OF INTEGERS MODULO N

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General area of research: Algebra and Its Applications, Graph Theory

Abstract:

A relatively recent and powerful tool to study algebraic rings is to construct graphs from them. In the past few decades, the introduction of the zero-divisor and total graphs of a ring has generated substantial results and interest in the literature. In this talk, we introduce a new class of graphs constructed from $\mathbb{Z}_n[i]$ using the property of whether the coefficients of the product of two elements are primes in $\mathbb{Z}$ or not. Those graphs turn out to have a surprising structure and provide a novel connection between commutative algebra, graph, and number theory.

Keywords: Graphs, Rings, Primes

2010 Mathematics Subject Classification: 05C25

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ZERO-DIVISOR GRAPHS OF SEMIRINGS

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Abstract:

The concept of zero-divisor graphs holds significant potential as a tool for understanding the algebraic structure of commutative rings. This paper extends the theory to semirings, a generalization of rings that relaxes the requirement for additive inverses. We explore the zero-divisor graph of a semiring S, where vertices correspond to non-zero zero-divisors of S, and edges represent pairs of distinct elements whose product is zero. By analyzing structural properties such as connectivity, diameter, and girth, we uncover significant differences and similarities between zero-divisor graphs of semirings and their ring counterparts. This work provides new insights into the interplay between algebraic properties of semirings and the characteristics of their zero-divisor graphs, opening pathways for further study.

Keywords: Zero-Divisor Graph, Semirings.

2010 Mathematics Subject Classification: 16Y60, 05C25

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MONOIDS OF MODULES

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General area of research: Commutative algebra

Abstract: Let R be a Noetherian local ring and M a finitely generated R-module. We let $\operatorname{add}(M)$ denote the monoid of isomorphism classes of modules that are direct summands of the direct sum of some finite number of copies of M. The monoid operation is given by the direct sum, that is, $[A] + [B] = [A \oplus B]$, where $[X]$ denotes the isomorphism class of the finitely generated module X. We characterize the monoids of the form $\operatorname{add}(M)$. That is, we describe the monoid-theoretic criteria shared by such monoids, and, given a monoid S satisfiying these criteria, we construct a local ring R and an R-module M such that $\operatorname{add}(M) \cong S$.

Keywords: Monoid, Local Ring, Module, Direct Sum

13C05, 13H99, 20M14, 2020 Mathematics Subject Classification:

POWER ORDER GRAPH OF A FINITE GROUP

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Abstract: The study of group structures can be performed in the setting of algebraic graph theory which is a powerful framework for studying such phenomena using graphical representations that allow visualization and deeper understanding of algebraic relationships. We often use graph forms to understand how group elements interact. These plots provide substantial information on subgroup structures, symmetries, and other group-theoretic characteristics.

In this work, we consider a new graph-theoretic notion called the Power Order Graph of a finite group G denoted by $P_o(G)$ and explore some basic properties. This graph introduces a novel adjacency condition that depends on the *power relationships* of group elements and the *divisibility of their orders*. Specifically, two elements a and b in G are adjacent if:

$b = a^n$, for some $n > 1$, where $|b| > 1$,and the order of b divides the order of a.

This definition is unique from classical power graphs because it incorporates the connectivity between element orders. So, it shows a clearer picture of the algebraic structure of a group. The Power Order graph depicts not just power relationships, but also the intra-group dynamics of orders contained within else order divisibility. On the contrary, for example in cyclic groups when the order of each element divides the group order, the graph is highly structured and symmetric. On the other hand, for non-abelian groups (like dihedral or quaternions), the graph reveals intricate interactions between rotational and reflective symmetries, leading to diverse connectivity patterns.

Keywords: Graphs on groups, Order of the element, subgroup.

2010 Mathematics Subject Classification: 05C25 , 05E16

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Generalized essential ideal graph of an N-group

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Abstract: Let N denote a zero-symmetric right nearring. We define the notion of g-essential ideal of an N-group. We introduce the notion of generalized essential ideal graphs and study the combinatorial properties such as connectivity, diameter, and completeness of this graph. Furthermore, when N is with 1 (the multiplicative identity) we consider matrix maps over N and establish the one-one correspondence between the g-essential ideals of a matrix nearring and that of the base nearring N.

Key Words: essential ideal, matrix nearring, N-group, graph.

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PRIME IDEALS IN NOETHERIAN RINGS

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General area of research: Commutative algebra

Abstract: We study partially ordered sets of prime ideals in Noetherian rings. In particular we give a general description of the partially ordered sets U that occur as Spec B, for some integral domain B that is a homomorphic image of a three-dimensional mixed polynomial-power series ring over a field or over a one-dimensional Noetherian integral domain. Most of this work is joint with Ela Celikbas, Christina Eubanks-Turner, William Heinzer, Christel Rotthaus, and Roger Wiegand.

Keywords: Prime ideals, Polynomials, Power series

13E05, 13F20, 13F25, 2010 Mathematics Subject Classification:

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Polynomial and power series rings over a generalized Krull ring

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Abstract:

Let R be a commutative ring with identity, X be an indeterminate over R, $R[X]$ be the polynomial ring over R, and $R[[X]]$ be the power series ring over R. In this talk, among other things, we show when $R[X]$ and $R[[X]]$ are generalized Krull rings.

Keywords: Generalized Krull ring, polynomial ring, power series ring

2010 Mathematics Subject Classification: 13A15, 13F05, 13F25

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SS8 - Statistical Learning and Data Science

SUPPORT VECTOR MACHINE CONTROL CHART FOR MULTIVARI DATA

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Abstract:

The emergence of quality 4.0, driven by industry 4.0 technologies, introduced complex, high-dimensional, and non-normal data that challenges the effectiveness of traditional control charts. Due to the abundance and diversity of quality 4.0 data, multivariate control charts (MVP) have been developed and can be divided into two categories: traditional and modern and machine learning (ML) based charts. Examples of traditional control charts are Hoteling T^2, Multivariate CUSUM (MCUSUM), and Multivariate Exponentially Weighted Moving Average (MEWMA) control charts, which require the data to be normally distributed, structured, and not correlated. Such charts use parameters such as mean, variance, sample size, and rigid control limits. Due to the shortcomings of traditional control charts, control charts that integrated ML methods such as Support Vector Machine (SVM), ANNs, and Decision Trees (DTs) were developed. SVM-based control charts

Although several studies proposed the use of SVM-based control chart, very few, if any, provided a way to handle one class data. Most of automated processes have rare out-of-control events. As a result, data collected in phase I will be in-control data. Such scenario will pose a challenge to develop the SVM hyperplane. The main objective of this study is to develop an SVM-based control chart for one class data

The objective of this study is to proposes a support vector machine-based control chart that can handle multi non-normal variables. The proposed Support Vector Machine (SVM) chart can also handle one class data which is very common in todays automated processes. The approach uses median rank nonparametric probability estimation to generate out-of-control vectors followed by kernel-based function hyper plane parameter estimation. The distance between vectors and hyperplane distance is used to develop the control chart and flag any out-of-control events. The proposed method is demonstrated using real life case study.

Keywords: Support vector machines, control charts, radial based kernels.

2010 Mathematics Subject Classification: Applied Statistics

A COMPOSITIONAL DATA ANALYSIS OF PHYSICAL ACTIVITY PATTERNS OF SCHOOL-AGED CHILDREN IN UAE

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Abstract:

The high prevalence of physical inactivity and obesity among school-aged children in the United Arab Emirates (UAE) has received notable attention over the past decade. Understanding the contribution of various school day segments to children's physical activity levels is essential for developing effective intervention strategies. This study aims to examine the physical activity levels (sedentary behavior [SB], light physical activity [LPA], moderate-to-vigorous physical activity [MVPA]) during different school day segments in Abu Dhabi. The school time spent in SB, LPA, and MVPA, expressed as a vector of nonnegative proportions that sum to one, is inherently compositional data. However, only recently have compositional data analysis (CoDA) techniques using Logratio models [1] been applied in time-use studies (e.g., [2, 3]).

For this study, 133 elementary school students from two schools in Abu Dhabi wore accelerometers for up to five non-consecutive school days [4]. Physical activity levels were measured across key segments, including class time, lunch, recess, physical education (PE), and the entire school day. Our analysis examines how biometric and demographic factors relate to the school time composition, employing Dirichlet regression, which is well-suited for compositional data [5].

Findings from this study will inform strategies to promote MVPA and optimize the distribution of movement behaviors throughout the school day, addressing inactivity and obesity among school-aged children in the UAE.

Keywords: School time, Compositional Data, Dirichlet Regression.

2010 Mathematics Subject Classification: 62P10

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Relative Range Sequential Approach for Detecting Outliers

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Abstract:

Outlier detection plays a key role in data analysis by improving data quality, uncovering data entry errors, and spotting unusual patterns, such as fraudulent activities. Choosing the right detection method is essential since some approaches may be too complex or ineffective depending on the dataset. In this study, we focus on introducing a new tool to sequentially detect outliers in univariate data based on relative range. In this work, we propose a new measure for detecting outliers in univariate data. The new measure, called relative range, is defined as the range statistic divided by the interquartile range (IQR). Since the range provides a simple yet effective measure of data dispersion, analyzing the range distribution will help identify potential outliers that fall outside the expected range of values. The probability distribution of the relative range is estimated for both symmetrical and skewed data distributions using Monte Carlo simulations. Based on the estimated empirical distribution of the relative range, a threshold is determined and used to detect potential outliers. This work proposes a sequential approach for outlier detection based on the relative range. In general, the relative range has shown to be a more robust statistic at detecting outliers in both sequential and non-sequential outlier detection.

Keywords: Outlier Detection, Range Distribution, Boxplot, Data Analysis

2010 Mathematics Subject Classification: Statistics and Probability

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LIFECYCLE-BASED ENHANCEMENT OF PHYSICAL ASSET MANAGEMENT IN UTILITIES

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Abstract:

The lifecycle of physical assets in utilities, encompassing planning, acquisition, operation, maintenance, and disposal, must incorporate certain considerations to sustain asset integrity and optimize efficiency. This study examines the performance of physical asset management (PAM) in utilities such as electricity, water, and wastewater. The aim of the study is to assess the impact of each asset lifecycle phase on utility performance and to identify enablers for physical assets inside utility organizations through Structural Equation Modelling (SEM). A maturity model was developed to evaluate asset management (AM) performance and recommend actions for improvement. The findings showed that all stages contribute positively to AM performance and underscored the critical role of effective planning in influencing on all subsequent phases. To address gaps in existing standards and frameworks, a tailored framework integrated with a maturity model was developed. This framework provides a comprehensive assessment tool to evaluate utilities' performance and deliver actionable recommendations for decision-makers to enhance performance. The maturity model was validated using five case studies for utility organizations. The proposed maturity model aid decision makers to assess and enhance current performance which has an important impact on the public wellbeing. The findings are valuable for decision-makers who seek improvement of utilities asset performance.

Keywords: asset management; utilities; asset lifecycle

A Non-Invasive Diagnostic Tool and Risk Calculator for Accurate Diagnosis of Thyroid Disorders

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Abstract:

Thyroid disorders, particularly hypothyroidism and hyperthyroidism, are prevalent endocrine conditions that are accompanied by severe consequences if not diagnosed accurately and in a timely manner. Current diagnostic approaches rely heavily on blood test measures. This study adopts a non-invasive approach utilizing data science tools to accurately diagnose hypothyroidism and hyperthyroidism. A TSH prediction model was first developed using 37 input features, including age, gender, family history, 32 disease related symptoms, and 2 extracted symptom scores for hypothyroidism and hyperthyroidism. A hybrid approach combining ANN, RF, and XGBoost was adopted, achieving an R^2 value of 94%. Next six machine learning models, including KNN, SVM, LR, DT, RF, and XGBoost, were employed to classify participants into normal versus non-normal thyroid status using the 37 input features and the predicted TSH levels. The results revealed that the XGBoost model was the best-performing model, with the highest F1 score of 99.44% and an accuracy of 99.84% compared to a 92% baseline accuracy. Subsequently, three subclassification models were developed for participants with non-normal thyroid status. The first model achieved perfect accuracy and an F1-score of 100% in distinguishing hypothyroidism from hyperthyroidism. The second model, which differentiated overt hypothyroidism from subclinical hypothyroidism, achieved a 90% F1-score and 90% accuracy, significantly higher than the baseline accuracy of 59.90%. The third model, designed to classify hyperthyroidism patients into subclinical versus overt class, demonstrated an F1-score of 90% and an accuracy of 94%, compared to a baseline accuracy of 82.35%. SHAP analysis was then employed to investigate the most important features of each model. The results revealed some variations in feature importance among different models, with overall agreement that hypothyroidism symptom scores and hyperthyroidism symptom scores were the most important features across all models, highlighting the significance of extracting features that aggregate the impact of symptoms on enhancing disease diagnosis. A risk calculator was then built using the developed models to serve as a screening tool for diagnosing hypothyroidism and hyperthyroidism in a cost-effective and more convenient manner, as it relies fully on a non-invasive approach. Such tools can also be used by individuals to monitor their thyroid status and by physicians to improve clinical decision-making. Furthermore, this study provides a robust framework for disease diagnosis, which can be implemented for other diseases and in different contexts.

A New Feature Selection Method for Qualitative Variables

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General area of research: Statistics

Abstract:

The increasing complexity of contemporary datasets, marked by high dimensionality and interdependencies, requires efficient feature selection techniques to improve the performance, computational efficiency, and interpretability of machine learning models. This research presents an innovative feature selection method used explicitly for qualitative variables utilizing the Relative Belief Ratio (RBR), a Bayesian statistical approach. The suggested method utilizes Bayesian principles to provide a strong alternative to conventional feature selection strategies, overcoming the constraints of frequentist approaches while ensuring computational efficiency. Conventional feature selection techniques are categorized into three types: filter, wrapper, and embedded methods. Filter approaches function independently from learning algorithms, assessing feature significance by statistical measures such as Chi-squared test, Information Gain, or Symmetric Uncertainty. Although computationally efficient, they may neglect interactions between characteristics. Wrapper methods, while effective, can experience high computational expenses and a tendency towards overfitting. Embedded approaches incorporate feature selection inside the model training process, achieving a balance between interpretability and computational requirements. Although these methods are useful, they predominantly focus on quantitative data, so neglecting the distinct issues associated with qualitative factors. This research presents a Relative Belief Ratio (RBR) based methodology that expands Bayesian statistics to assess feature significance [1]. In contrast to frequentist methods that depend on p-values and hypothesis testing, RBR evaluates the plausibility of hypotheses by synthesizing prior beliefs with actual facts. This method evaluates characteristics based on importance and quantifies the evidence supporting their selection. This strategy is practical at managing categorical data by utilizing the Dirichlet distribution to represent prior and posterior probability.

This method is fundamentally based on the computation of the RBR and its corresponding strength value. The RBR evaluates the posterior density of a hypothesis relative to its prior density, offering a dynamic framework for revising beliefs based on new data. Furthermore, the strength value quantifies the evidence validating the null hypothesis, providing a refined understanding of feature significance. Future research areas involve the application of the suggested method to bigger and more intricate datasets, evaluating its effectiveness in classification tasks, and confirming its performance through comparisons with existing methodologies such as Chi-squared test or Information Gain. The incorporation of RBR with machine learning methods, like Random Forests and Logistic Regression, will further evaluate its practical applicability. Comparative evaluations will examine its computational efficiency and predictive accuracy in relation to other methodologies. This research presents a significant methodological development in machine learning and data analysis by addressing the deficiencies in feature selection for qualitative data. The RBR-based methodology improves decision-making in feature selection, providing a scalable, interpretable, and economical resolution to issues with high-dimensional data.

Keywords: Feature selection, Relative Belief Ratio, Qualitative data

2010 Mathematics Subject Classification: 62F15

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REFINING BAYESIAN ANALYSES WITH IMPROPER PRIORS USING K-FOLD CROSS-VALIDATION

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Abstract: In this talk, we introduce a three-stage Bayesian updating framework for refining analyses that begin with an improper prior. The dataset is partitioned into K equally (or nearly equally) sized folds. The proposed methodology comprises three steps: (1) deriving interim posterior distributions for each fold in a cross-validation process using the improper prior, (2) updating these interim posteriors with data from $K-2$ folds to produce final posterior distributions, and (3) utilizing the remaining fold to evaluate and select the optimal model based on validation performance, measured by the mean squared error. This framework is general and adaptable to a wide range of likelihood models, providing a systematic approach to improving Bayesian inference with improper priors. As a practical application, we illustrate the method in the context of a hypothesis testing problem.

Keywords: Cross-validation, Improper priors, Model selection.

2010 Mathematics Subject Classification: 62F15, 62G09

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Estimation of Multiple Linear Regression Model Parameters under Multicollinearity: A Comparative Study of New Ridge-Type Pretest and Shrinkage Estimators with Machine Learning Approaches. An Application to Marsh Grass Biomass.

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Abstract:

Multicollinearity is a widespread problem in regression analysis that arises when some predictor variables exhibit high correlation, resulting in unstable least squares estimations of the model parameters. Various estimation strategies were used to mitigate the impact of this issue. This article enhances a new ridge-type estimator by using the pretest and shrinkage estimation techniques. An analytical comparison was conducted to assess the effectiveness of the proposed estimators based on their biases, quadratic risks, and numerical evaluations using simulated and real data examples. Additionally, we evaluated multiple penalization techniques and three machine-learning algorithms to facilitate comparison. Our results demonstrated that the proposed estimators outperformed the new ridge-type estimator in terms of the mean squared error for simulated data and the mean squared prediction error for the real dataexampl

Keywords: Ridge-Type estimator, pretest, shrinkage, penalization methods

2010 Mathematics Subject Classification: 62J07, 62F10, 62C20, 90C25

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ACCOMMODATING SPATIAL OUTLIERS IN SPATIAL REGRESSION MODEL WITH APPLICATIONS ON COVID-19 DATA

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Abstract:

Spatial Outlier is an observation that displays a noticeably different characteristic from its neighborhood observations. Researchers in spatial statistics believe that such observations reserves important information that call for attention in applications. Accommodating outlier in traditional statistics is a common phenomenon. However, in spatial regression model settings, accommodating spatial outlier has not adequately been addressed. This work develops a technique that successfully accommodate spatial outliers in Spatial Regression Model. Restricted Maximum likelihood is adopted on spatial regression model with variance shift model. Variance Shift Model is a mixed effect model which has a random effect which detects outlier values and then accommodate them into the model with the assumption that there is a shift in the variance of the outlier observation. The spatial variance model entails accommodating spatial outliers based on weights assigned to detected outliers in the spatial variance shift model. Simulation results have shown remarkable performance in accommodating the spatial outliers. Georgia state Covid-19 data were employed to demonstrate the performance of the method.

Keywords: Spatial Outlier, Spatial Regression Model, Variance Shift Model.

2010 Mathematics Subject Classification:

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SS9 - Statistics and Data Science for Digital Finance on Tokenomics

DİGİTAL İNNOVATİONS' CONTRİBUTİON TO REDUCİNG THE SHADOW ECONOMY İN EMERGİNG EU COUNTRİES

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Abstract:

Digital transformation has emerged as a driving force in reshaping tax systems and global economic practices. This study addresses the implications of digitization in reducing tax avoidance in the European Union (EU) Member States, where the time frame of analysis spans a 9-year period between 2013 and 2022. Thus the aim of this research is to highlight some correlations between tax avoidance and digitization. The means used for this cross-sectional and temporal dataset are based on the application of a regression on panel data, where we used the dependent variable tax evasion represented by the shadow economy and the Digital Economy and Society Index as independent variable. The study extends the literature by comparing the impact of digitization on the shadow economy in developing and developed countries and by introducing a dummy variable on the sense of happiness, with the intention of showing whether people are happier when they pay less taxes. The research therefore demonstrates how the implementation of digital infrastructure, digitization of public services, and the adoption of electronic payment technologies have influenced the levels of the shadow economy. The results provide evidence that increased digitization in developed countries leads to a reduction in the shadow economy through improved tax collection tools. Compared to developing countries where the linkage intensity is significantly lower. Finally, this study provides insights that contribute to the design of effective tax policies to increase tax transparency in countries at high risk of tax avoidance.

Keywords: tax avoidance, shadow economy, digitalization.

2010 Mathematics Subject Classification: 62G10, 62J05.

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STYLIZED FACTS OF METAVERSE NON-FUNGIBLE TOKENS

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General area of research: Statistics

Abstract:

Non-Fungible Tokens (NFTs) within the metaverse represent a rapidly emerging sector in the digital asset space. This paper provides a comprehensive review of the metaverse's history and an analysis of the stylized facts of five metaverse NFTs: Axie Infinity, Decentraland, Enjin Coin, Theta Network, and The Sandbox. We examine market efficiency, volatility clustering, leverage effects, and the return-volume relationship. Our key findings show that all NFT returns exhibit kurtosis values significantly exceeding the standard value of three, indicating more peaked and heavier-tailed distributions than a normal distribution. Autocorrelation analysis reveals statistically insignificant results, suggesting minimal influence of past returns on current returns. The Hurst exponent fluctuates between 0.3 and 0.8, indicating relative inefficiency in log returns with varying degrees of trend reinforcement and anti-persistence. The GARCH(1,1) model confirms the presence of volatility clustering, with high persistence of volatility shocks over time, and most NFT returns exhibit a negative leverage effect, where negative returns decrease volatility. These findings provide critical insights for investors, content creators, and policymakers, emphasizing the need for innovative strategies and regulatory considerations in this evolving ecosystem. A comparative analysis using alternative metaverse-related assets from Bloomberg and Yield Guild Games enhances the robustness of our findings, enriching the academic discourse on digital assets and laying the groundwork for future research in metaverse NFTs.

Keywords: Metaverse; Non-fungible tokens; Stylized facts

2010 Mathematics Subject Classification: 62M10, 62G32, 62H20

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DIGITAL ASSETS IN WAR - A DOUBLE-EDGED SWORD?

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General area of research: Statistics

Abstract:

Over the past decade, digital assets have been exposed to a wide range of significant global events ranging. Before 2022, digital assets had never witnessed a major military conflict and simultaneously played a significant role. This all changed with the Russia-Ukraine and Israel-Hamas conflicts in 2022 and 2023, respectively. A key question is how have these conflicts impacted digital asset markets? In this talk, we will analyse this impact from two financial perspectives, discussing some recent work investigating the short-term impact of conflicts on high frequency digital asset markets through an event study approach, and analysing the evolution of digital asset network structures before and after conflicts and their relationships with key economic factors. Our event study analysis addresses a number of drawbacks in existing studies [1], by considering the wider digital asset market, the high frequency impact of military conflicts, and specifically positive and negative events beyond just the initial invasion. The results suggest that negative war-related events have both an immediate and sustained impact on cryptocurrencies and DeFi tokens in the short term, which may be due to a series of negative events leading to an accumulation of positive returns. Overall, we see a contrast to the "negativity effect" of war-related events on stock markets [2], which suggests that investors may gain from switching to cryptocurrencies in times of conflict. Our analysis of the digital asset networks builds on similar studies relating to COVID-10 [3], and reveals that prior to the official start of both wars, the networks are highly interconnected, likely due to investor anticipation of conflict. However, after the wars begin, the networks become significantly fragmented, with distinct differences in structure between the Russia-Ukraine and Israel-Hamas conflicts, influenced by market reactions and varying levels of global attention. The time-varying analysis shows that key economic variables like geopolitical risk, the U.S. Dollar Index, and oil volatility significantly influence the digital asset networks. While geopolitical risk and oil volatility generally increase network centrality and density, the U.S. Dollar Index exhibits a sharp negative impact during the Russia-Ukraine conflict, and broader economic factors, such as Federal Reserve interest rate hikes, further shape network dynamics post-2022.

Keywords: Cryptocurrency; Network; Event Study

2010 Mathematics Subject Classification: 62-07

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Reaction Times to Economic News in High-Frequency Trading: An Analysis of Latency and Informed trading ahead of macro-news announcements

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Abstract:

This paper conducts an event study to analyze high-frequency trading (HFT) reaction times to macroeconomic news releases on CME and Eurex, with a focus on the higher precision timestamps available at Eurex. Utilizing high-precision timestamp data and latency measurements, we examine the immediate market response to announcements such as PMI (ISM Manufacturing, ISM Services), Non-Farm Payrolls (NFP), and Federal Open Market Committee (FOMC) releases. Our analysis reveals significant discrepancies when accounting for latency, suggesting potential pre-release access to data. Hypotheses are proposed regarding the distribution mechanisms of macroeconomic data to global providers. Robustness checks confirm that these latency discrepancies are absent during non-macro data release periods, reinforcing the specificity of the observed effect.

Keywords: High-frequency trading, Market microstructure, Economic news releases

2010 Mathematics Subject Classification:

THE IMPACT OF CORPORATE GOVERNANCE ON THE ENVOLMENT IN CORPORATE SOCİAL RESPONSİBİLİTY ACTIVITIES

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Abstract:

Corporate social responsibility (CSR) reflects ethics and corporate actions on three pillars, reflecting on environmental reporting, social features and governance (ESG). CSR is a key factor found in the literature for achieving economic goals and creating wealth.

Companies with higher levels of governance offer greater transparency in the implementation of daily rules and operations. They can make timely adjustments to existing problems in the operation of the company. At the same time, they can avoid petential risks over time, reduce transaction costs and improve the efficiency of production and operation.

Regarding the methodology, the data were collected using the Thomson Reuters database, and their processing was done using the Stata 15 statistical threshold. The study was conducted over a ten year period (2013-2023) on the states of the European Union.

The results of our research capture a strong bond between high corporate governance and engagement in CSR activities.

Our research is of interest to companies that currently do not have the obligation to report ESG activities. The study can also impact stakeholders (banking companies, investors, clients, the state).

The participation of companies in social responsibility activities is beneficial for establishing trust between the company and stakeholders and can also strengthen the sense of responsibility of employees towards companies. The established relationship is a strong point of the companies and also helps them to understand their shortcomings in the strategic objectives to be developed.

Keywords: governance, CSR, ESG.

2010 Mathematics Subject Classification:
- o 62-07 Data analysis;
- o 62F03 Hypothesis testing;
- o 62J05 Linear regression.

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PREDICTING FINANCIAL TRENDS USING TEXT MINING AND NLP

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Abstract:

This research aims to build predictive models for financial trends, combining artificial intelligence with natural languange processing to improve forecasting accuracy in financial risk domains. The primary aim is to advance models that assess credit risk, detect fraud, and predict compliance issues by incorporating sentiment analysis from unstructured text data such as news articles, social media posts, and audit reports. In particular, how to build strategies for integrating ESG reports to transparently disclose environmental, social, and governance risks that may impact a company's creditworthiness. The secondary aim of the project is related to the improvement of supervisory tasks of central banks. By combining structured data (e.g., debt ratios) with insights derived from reports and social media data, central banks can build stress-testing scenarios that account for ESG-related risks, capital risks and liquidity risks, such as environmental fines or social compliance costs. When conducting stress tests, NLP can analyze the narrative sections of stress-testing reports from banks to identify potential vulnerabilities not captured in quantitative data alone. This allows central banks to adjust their assumptions based on qualitative insights.

Keywords: NLP, Digital Finance

2010 Mathematics Subject Classification: 91G70

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Ethereum Blockchain Analytics: What We Learn on Token Price from Geometric Deep Learning Models

Yuzhou Chen

In the last few years, Geometric Deep Learning (GDL), e.g., Graph Neural Networks (GNNs), have emerged as a powerful alternative to more conventional deep learning (DL), machine learning (ML), and statistical models from clustering/anomaly detection to node classification/link prediction to time-series forecasting. Despite their proven success, GNNs tend to be limited in their ability to simultaneously infer latent temporal relations and encode higher-order interactions among entities. Our works tackle these limitations across a spectrum of higher-order network analysis, from topological data analysis to simplicial complexes. In our works, we propose a novel time-conditioned topological representation, and make the first step on a path of bridging the two emerging directions, namely, time-aware GDL with time-conditioned topological representations with applications on Ethereum blockchain networks and beyond.

DOES THE İNTERPLAY BETWEEN POLLUTİON AND ECONOMİC CONNECTİVİTY İMPACT HAPPİNESS? EVİDENCE FROM EUROPEAN COUNTRİES

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Abstract: A clean environment is essential for a sustainable economy and a happy society. From this perspective, our study assesses the interaction effect of pollution and economic connectivity (EC) on well-being in EU countries. Dynamic panel fixed-effects estimates reveal that the CO2 impact on happiness worsens as EC increases, suggesting that the states have not yet reached the threshold in EC that promotes an ecologically sound development. Thus, for Europeans' happiness to develop in a sustainable environment, decision-makers should be more attentive to economic-financial evolution in their undesired environmental effects.

Keywords: happiness-pollution, economic connectivity, dynamic panel models.

Acknowledgments: This work was supported by a grant from Babeș-Bolyai University for early-career researchers, project number SRG-UBB 32868/20.06.2023, period 2023-2024 project manager Gabriela-Mihaela Muresan. Also, Alexandra-Anca Purcel and Codruta Mare acknowledge that this work was supported by the project "A better understanding of socio-economic systems using quantitative methods from Physics" funded by the European Union-NextgenerationEU and the Romanian Government, under National Recovery and Resilience Plan for Romania, contract no 760034/23.05.2023, cod PNRR-C9-I8-CF255/29.11.2022, through the Romanian Ministry of Research, Innovation and Digitalization, within Component 9, Investment I8. Authors are listed in alphabetical order, and the usual disclaimer applies.

2010 Mathematics Subject Classification: 97K80.

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GENERAL SESSIONS

GS1 – Algebra, Number Theory & Applications

Further Results Related to the Numerical Radius of Block Matrices

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Abstract. This paper aims to provide a new bound to the numerical radius of the off-diagonal part of positive semidefinite block matrices in terms of the diagonal blocks. It is proven that if $X, Y, Z \in M_n(\mathbb{C})$ are such that $A = \begin{bmatrix} X & Z \\ Z^* & Y \end{bmatrix} \geq 0, A \neq 0$. Then

$$w^{2s}(Z) \leq \left(\frac{\lambda - \mu}{\lambda + \mu}\right)^{2s} \left\| \beta X^{\frac{s}{\beta}} + (1-\beta) Y^{\frac{s}{1-\beta}} \right\|,$$

where λ, μ are the largest and smallest eigenvalues of A, $0 < \beta < 1$ and $s \geq 1$.

2010 Mathematics Subject Classification. 47A12, 47A30, 47A63.

Keywords. Positive semidefinite matrix, operator norm, numerical radius, block matrix.

NONSPEEDABLE SETS AND Q_1-REDUCIBILITY

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Abstract:

Suppose that $\{\varphi_i\}_{i\in\omega}$ is an effective numbering of all partial computable functions and let $\{\Phi_i\}_{i\in\omega}$ be a sequence of partial computable functions satisfying the following Blum's axioms [1]:
(1) the function $\varphi_i(x)$ is defined if and only if the function $\Phi_i(x)$ is defined;
(2) the function

$$M(i,n,m) = \begin{cases} 1, & \text{if } \Phi_i(n) = m, \\ 0, & \text{otherwise,} \end{cases}$$

is computable.

According to [2] a c.e. set A is called speedable if for all i such that $W_i = A$ and for all computable functions h, there exists j such that $W_j = A$ and

$$(\exists^{\infty} x)(\Phi_i(x) > h(x, \Phi_j(x))),$$

and A is called nonspeedable otherwise.

We say that a set A is Q_1-reducible to a set B (in symbols: $A \leqslant_{Q_1} B$) if there exists a computable function f such that for every $x \in \omega$ (where ω denotes the set of natural numbers),

$$x \in A \Longleftrightarrow W_{f(x)} \subseteq B,$$

and for all x, y,

$$x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \varnothing.$$

In [3] it is proved that the c.e. Q_1-degrees are not dense and there exists a non-empty open interval of Q_1-degrees of speedable sets.

In this talk we will present some recently obtained results about nonspeedable sets and Q_1-reducibility. In particular, we prove that if A and B are noncomputable c.e. nonspeedable sets such that B is not a simple set, $A <_{Q_1} B$ and $A \oplus B \leqslant_{Q_1} B$, then there exist infinitely many pairwise Q_1-incomparable c.e. sets $\{C_i\}_{i\in\omega}$ such that for all $i \in \omega$, $A <_{Q_1} C_i <_{Q_1} B$.

Keywords: Nonspeedable set, Q_1-reducibility.

2010 Mathematics Subject Classification: 03D25, 03D30

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EXPLORING KAPREKAR'S ALGORITHM IN ALTERNATIVE DIGIT ARRANGEMENTS

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General area of research: Number Theory

Abstract:

Let $x \in \mathbb{Z}$ represent any n-digit integer with some arrangement of the digits $\{x_n, x_{n-1}, ..., x_1\}$ such that $x_n \geqslant x_{n-1} \geqslant ... \geqslant x_1$ and $x_n > x_1$. Sort the digits in descending and ascending order where $x_{desc} = x_n x_{n-1} ... x_1$ and $x_{asc} = x_1 ... x_{n-1} x_n$. Define a function $K(x) = x_{desc} - x_{asc}$ and apply it to x. Repeat this routine using the result of $K(x)$ as the new input. In 1949, Kaprekar discovered that this iterative process, known as Kaprekar's Algorithm, always converges to 6174 within 7 iterations for any 4-digit x [1]. Mathematical proofs of the convergence of Kaprekar's Algorithm are presented for the 3-digit and 4-digit case, where any 3-digit integer converges to 495 within 6 iterations. Additionally, a generalized computational framework is built to explore the behavior and convergence of Kaprekar's Algorithm for any n-digit integer in any base $b \leq 10$.

Keywords: Kaprekar's Algorithm, Convergence

2010 Mathematics Subject Classification:

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GS2 – Analysis

A mathematical analysis of a system of caputo-Fabrizio fractional differential equations for cancer combination treatment

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Abstract: Mathematical modeling has become an important and useful tool in studying the interactions between the immune system and a growing tumor. In this paper, we generalize the classical system to a new fractional system named the caputo-Fabrizio derivative and we check the stability of the systems in the three cases, single immunotherapy, single chemotherapy and mixed treatment.

$$\begin{aligned}
\frac{dx}{dt} &= s + \frac{\rho xy}{\alpha + y} - c_1 xy - d_1 \cdot x - \alpha_1 \cdot (1 - e^{-z})x \qquad t \neq n\tau \\
\frac{dy}{dt} &= ry(1 - by) - c_2 xy - \alpha_2 (1 - e^{-z})y \qquad t \neq n\tau \\
\frac{dz}{dt} &- d_2 z \qquad t \neq nt \\
x(t^t) &= x(t) + \mu_1 \qquad t = n\tau \;\; y(t^+) - y(t) \;\; t = n\tau \;\; z(t^t) = z(t) + \mu_2 \; \tau = n\tau
\end{aligned}$$

where x denotes the concentration of CTL cells with antitumor activity in the tumor site, y represents the number of tumor cells, and z is the blood drug concentration. τ is the therapeutic period, μ_1 is the infusion dose of CTL cells with antitumor activity every time, and μ_2 denotes an increment of the blood drug concentration due to delivering drug at time $t = n\tau.$, $x(t^+), y(t^+)$ and $z(t^+)$ denote the right limits of $x(t), y(t)$ and $z(t)$ at time t respectively.

Keywords: Immunotherapy, Caputo-fabrizio derivative (CF-system), Laplas adomian decomposition method (LADM).

2010 Mathematics Subject Classification: 34A07; 26A33

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RATE OF APPROXIMATION BY GENERALIZED SAMPLING OPERATORS

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Abstract:

In this paper, we examines the approximation properties of generalized sampling operators in the settings of $C(\mathbb{R}^d)$. We obtain the rate of approximation in-terms of the modulus of continuity for the generalized sampling series for functions in $C(\mathbb{R}^d)$. Finally, we estimate the rate of approximation for these sampling operators under the condition of Hölder continuity.

Keywords: Sampling operator; Direct estimate; Modulus of smoothness.

2010 Mathematics Subject Classification: Primary 26A15; Secondary 41A25.

Generalized Mittag-Leffler-Confluent Hypergeometric Functions in Fractional Calculus with Integral Operator

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Abstract:

Originally, confluent hypergeometric functions and Mittag-Leffler functions were developed to approximate interpolation in the exponential function. The purpose of this work is to investigate the operators of generalized Mittag-Leffler-type functions. Under certain unusual circumstances, the generalized Fox-Wright function will be utilized. Furthermore, we will investigate some of the commonly employed generalized fractional integral operators in fractional calculus. Researchers have recently learned more about generalized Mittag-Leffler-type functions by using generalized fractional differential and integral operators. The study's results add to this wealth of knowledge. There will also be a thorough analysis of the various implications and effects of these results.**Keywords:** state a maximum of three terms.

2010 Mathematics Subject Classification: Primary: 26A33; 33C60; 33E12. Secondary: 33E20; 45J05.

Keywords: Laplace transform; Fractional calculus; Confluent hypergeometric function; Mittag-Leffler function; Integral operator.

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BOUNDEDNESS OF COUPLED PSEUDO-DIFFERENTIAL OPERATORS ON SCHWARTZ SPACE

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Abstract:

In this talk, we obtained some fruitful results of the coupled fractional Fourier transform and its kernel. We defined pseudo-differential operators related to coupled fractional Fourier transform on Schwartz spaces and it is shown that their composition is again a pseudo-differential operator. It made further discussion on the composition of two pseudo-differential operators on Sobolev spaces and derived certain norm inequalities on it. Further, we have successfully applied some of the results of the coupled fractional Fourier transform to investigate the solution of n^{th} order linear non-homogeneous partial differential equation and wave equation. Lastly, we present some examples involving graphs and tables to illustrate the validity of our theoretical findings.

Keywords: Fractional Fourier Transform, Coupled Fractional Fourier Transform, Pseudo-differential Operator.

2010 Mathematics Subject Classification: 42B10, 44A15, 35S05, 46F12, 46E35

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QUASI HYPERHARMONIC FUNCTIONS IN AXIOMATIC HARMONIC SPACES

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General area of research: Analysis

Abstract: We introduce the concept of quasi-hyperharmonic functions in axiomatic Harmonic spaces and then we give a characterization of such functions in terms of different classes of measures. We show that a lower semicontinuous regularization $\hat{u}$ of a quasi-hyperharmonic function u is in fact hyperharmonic. Moreover, we show that $u = \widehat{u}$ out side of a negligible set. The concept of quasi hyperharmonic function will be extended to a product of harmonic spaces.

Keywords: Brelot harmonic space, harmonic functions, hyperharmonic functions..
2010 Mathematics Subject Classification: Primary 31B05; 31B15, 31D05, 32U20

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GS3 - Applied Mathematics & Differential Equations

STABILITY ANALYSIS OF CHARGED NEUTRON STARS AND DARMOIS JUNCTION CONDITIONS

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ABSTRACT

This research article examines the impact of $f(Q, T)$ theory on the geometry of charged neutron stars filled with anisotropic matter configuration. Here, Q represents non-metricity and T denotes the trace of energy momentum tensor. We use a particular functional form of this modified theory to reduce the system's complexity and derive explicit relations of the energy density and pressure components. Further, we consider viable non-singular solutions to analyze the internal structure of the charged neutron stars. The unspecified parameters in the metric coefficients are evaluated through Darmois junction conditions, which ensures consistency between interior and exterior solutions of the stellar objects. These parameters are then used to explore different physical characteristics such as the behavior of energy density, pressure components, anisotropy, energy bounds, equation of state parameter, compactness and redshift function in the interior of charged neutron stars. The stability and equilibrium states of the charged stellar objects are discussed using the Tolman–Oppenheimer–Volkoff equation and the speed of sound, respectively. Our results suggest that the charged neutron stars are viable and stable in the presence of dark source terms.

Key Words:

Charged Neutron Stars, Stellar Stability, Anisotropic Configuration, Energy-Momentum Tensor (T)

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STUDY ON MASS LOADING SENSITIVITY IN BUFFER LAYERED PIEZO-MAGNETIC FIBER-REINFORCED COMPOSITE STRUCTURE ANALYSING SHEAR WAVE PROPAGATION

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Abstract: This study is motivated by the need to investigate the untapped propagation characteristics of Love-type waves in smart material layered structures with varied geometrical complexities, which offer the potential for improved multi-field coupling effects. In this theoretical study, we examine the behaviour of Love-type waves within a layered structure composed of piezo-magnetic fiber-reinforced material with a thin-coated mass loading on the top surface. Additionally, a buffer layer of functionally graded piezo-magnetic (FGPM) material is positioned between the layer and the half-space. By employing the power series method and solving the systems of equations, closed-form velocity equations have been obtained. Our analysis explores the influence of FGPM layer thickness and mass loading on key factors, including dispersion relations, magneto-mechanical coupling factor and mass loading sensitivity of Love-type waves in this configuration under magnetically open and short conditions. The theoretical insights from this study provide valuable guidance for designing high-performance surface acoustic wave devices using FGPM buffer layers and suggest potential methods for measuring material properties in FGPM layered systems with Love-type waves.

Keywords: Piezo-magnetic fiber-reinforced, Buffer Layer, Mass loading sensitivity.

2010 Mathematics Subject Classification: 35Q74, 74A05, 74A10, 74B05, 74F15, 74E05, 74E10, 74E15, 74E30, 74K35, 74H10, 74J15

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OPTMIZING PERMEABILITY DOMAIN FOR HYDROGEN FUEL CELLS

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Applied Mathematics

Abstract:

Hydrogen fuel cells (HFCs) are becoming an important technology for cleaner energy. They produce electricity through chemical reactions without harmful emissions, making them useful in many areas like powering cars, and buses, and providing energy in remote locations. As the world moves towards more sustainable energy, improving the efficiency of fuel cells is essential. In this paper, we introduce the effect of changing the permeability function inside the Gas Diffusion Layer (GDL), which is in the middle of a HFC, and how it affects the flow of fluids inside the cell. Permeability controls how gases move through the fuel cell, impacting many variables. Understanding these effects is important for making fuel cells work better. Using FreeFEM++, we numerically solve the nonlinear system of partial differential equations derived from the convection-diffusion equation and the Darcy law, using the finite element method, considering three essential variables: oxygen concentration, pressure, and velocity. We will also present an optimum permeability function that helps maximize efficiency.

Keywords: Hydrogen Fuel Cells, Finite Element Method , Optimization

Partial differential equations

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SOLVING THE FULLY FUZZY RICCATI MATRIX EQUATION IN LINEAR QUADRATIC REGULATOR PROBLEMS USING GRADIENT AND LEAST SQUARE ITERATIVE METHODS

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Abstract:

A Linear Quadratic Regulator (LQR) is a fundamental control problem used in various engineering disciplines, including robotics, aerospace, and economics. The solution of the LQR problem often involves solving a Riccati matrix equation, which is a crucial step in determining optimal control policies. However, in many real-world applications, the system parameters are not precisely known and uncertainty arises in the form of fuzzy data. This research proposes a novel method for solving the fully fuzzy Riccati matrix equation in LQR problems using gradient and least-squares iterative methods. The goal is to develop a robust approach to solve Riccati equations under uncertainty while maintaining the optimality of the control policy. This paper explores the formulation of the fuzzy Riccati equation, presents solution techniques using gradient and least-squares methods, and analyzes the application of fuzzy logic in the context of LQR.

Keywords: Fuzzy Logic; Linear Quadratic Regulator ; Riccati Matrix Equation

2010 Mathematics Subject Classification: 03B52; 94D05

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THE PRICE OF COGNITION AND EVOLUTIONARY DYNAMICS OF LEARNING IN NEURAL NETWORKS THROUGH WARDROP'S EQUILIBRIA APPROACHES

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General area of research: Applied Mathematics, Mathematical Biology

Abstract:

Neurotransmission is a fundamental physiological mechanism that facilitates each cognitive process. In a broader context, the experimental investigations postulate that toxic neuropeptides/neurotransmitters are causing damage to the functionality of synapses during neurotransmission processes. In this talk, we propose a novel mathematical model that studies the dynamics of synaptic damage in terms of concentrations of toxic neuropeptides/neurotransmitters during neurotransmission processes. Our primary objective is to gain insight into specific mechanisms and phenomena within neural networks by applying models and methods originally designed for solving optimal flow distribution problems in transportation networks. The degradation of the functionality of synapses occurring due to elevated levels of toxic neuropeptides/neurotransmitters during the neurotransmission process can be linked to congestion games in transportation networks. For this purpose, we employ Wardrop's first and second principles within a neural network of the brain. Although these principles were originally formulated to describe optimal flow distributions in transportation networks, we adapt and apply effectively these principles within the framework of the neural network, which is the novel approach.

In order to incorporate Wardrop's first and second principles into the neural network of the brain, we introduce two novel concepts: *neuropeptide's (neurotransmitter's) equilibrium* and *synapses optimum*. The *neuropeptide/neurotransmitter equilibrium* refers to *a distribution of toxic neuropeptides/neurotransmitters that leads to uniform damage across all synaptic links*. Meanwhile, *synapses optimum* is *the most desirable distribution of toxic neuropeptides/neurotransmitters that minimizes the cumulative damage experienced by all synapses*. In the context of the brain neural network, an analogue of the price of anarchy is *the price of cognition* which is *the most unfavorable ratio between the overall impairment caused by toxic neuropeptide's (neurotransmitter's) equilibrium in comparison to the optimal state of synapses (synapses optimum)*.

Additionally, a discrete-time replicator equation is proposed within this framework that leads to the establishment of the synapses optimum during the neurotransmission process. Using the Lyapunov functions, we study the stability of the rest points of the corresponding mean-field dynamics. We propose an iterative neurodynamics and learning algorithms using replicator equations, in which the synapses optimum is reached during the neurotransmission process. Finally, we discuss some computational experiments.

Keywords: Neural network, Wardrop's equilibria, learning dynamics, replicator equation

2010 Mathematics Subject Classification: 92B20, 91A26, 91A22, 37N25

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DISCRETE-TIME REPLICATOR DYNAMICAL SYSTEMS ON WARDROP-SCHUR OPTIMAL TRANSPORTATION NETWORKS

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General area of research: Applied Mathematics, Analysis

Abstract:

In this talk, we explore a novel application of replicator dynamics by introducing discrete-time replicator equations as a framework for analyzing optimal transport networks with congestion. We begin by defining the concept of a Wardrop optimal network, characterized by Wardrop optimal flows that simultaneously satisfy the conditions of Wardrop-Nash equilibrium and system optimum. These networks uniquely achieve a price of anarchy equal to its least value of 1, which is a highly desirable property.

Based on this, we propose a new dynamical model for optimal flow distribution on Wardrop optimal networks, grounded in the principles of evolutionary game theory. Unlike classical game theory, evolutionary game theory emphasizes the dynamics of strategy change and adaptation over time. Our model employs discrete-time mean-field replicator equations formulated over probability simplices and generated by nonlinear similar order-preserving mappings. Specifically, we investigate replicator dynamics generated by convex differentiable functions and Schur-convex potential functions, where in the latter case the networks introduced are called Wardrop-Schur optimal networks. As illustrative examples, we utilize complete symmetric functions, gamma functions, and symmetric gauge functions of unitary invariant norms that generate replicator dynamics on Wardrop-Schur optimal networks.

The analysis focuses on the dynamic properties of replicator systems, with particular emphasis on convergence and stability. Using tools from dynamical systems theory, including Lyapunov functions, we rigorously examine Wardrop-Nash equilibria, convergence to fixed points, and conditions for asymptotic stability. Notably, for the replicator equations under study, the Nash equilibrium, the Wardrop equilibrium, and the system optimum are shown to coincide, corresponding to the same state in the flow space.

Additionally, we present methods for constructing Wardrop optimal networks, including affine and nonlinear transformations that preserve Wardrop optimality, as well as stochastic approaches to Wardrop optimal network design. These results contribute to a deeper understanding of applications of replicator equation dynamics in the context of optimal transport and network optimization.

Keywords: Evolutionary game dynamics, optimal transport on networks, Schur-convex functions

2010 Mathematics Subject Classification: 37N40, 91A22, 90C33, 49Q22, 90B10

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ON THE EUCLIDEAN DISTANCE TO INSTABILITY

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General area of research: Applied Mathematics

Abstract: The distance to instability of a matrix is a robust measure for the stability of the corresponding dynamical system, known to be far more reliable than determining the position of matrix eigenvalues. We will present new estimates for such a distance, for some special types of matrices. Computationally the most expensive part involves checking if a given matrix belongs to the particular class. However, the conditions that have to be checked depend on matrix entries only, so they can be considered as relatively cheap. In order to estimate the distance to instability, we will use good estimations of the norm of the inverse for matrices belonging to special subclasses of H-matrices. The matrix which we will consider as an example comes from a model of energy flow in soil food webs (for a detailed treatment of this problem, see [1, 2]). The model is formulated as the Lotka-Volterra predator-prey system of nonlinear differential equations, so the community matrix and its properties are essential for dynamical analysis, including asymptotic stability, then robust stability, and finally transient behavior. Since all diagonal elements of such community matrices are negative, stability is ensured knowing that the community matrix belongs to the class of H-matrices. Due to computational costs, we never check this property by definition, but check whether the community matrix belongs to some subclass of H-matrices. Most of subclasses are based on the maximum matrix norm (for an overview of such subclasses, see [3]), which can realistically be interpreted as conditions of trophic influences within one functional group. Due to uncertainties in empirical data, these conditions are often violated. If we use the Frobenius norm, instead, i.e. an approach that treats trophic influences between all functional groups together, we will get a condition more robust to the aforementioned uncertainties. Because of that we will refer to the recent subclass of H-matrices introduced in [4], which is related to Euclidean vector and Frobenius matrix norm. Additionally, we will single out two particular subclasses, for which it is possible to discuss distance to instability and pseudospectral properties easily, very much like it has been done in [5] by Geršgorin and pseudo-Geršgorin set.

Keywords: H-matrices, distance to instability, dynamical systems.

2010 Mathematics Subject Classification: 15A12, 15A18, 15B99, 65F99

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GRONWALL TYPE INEQUALITIES FOR NORMALIZED FRACTIONAL INTEGRALS WITH MITTAG-LEFFLER KERNELS WITH APPLICATIONS

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Abstract:

Gronwall inequalities are essential tools in studying various types of differential and integral operators. They are commonly employed to derive results related to existence, uniqueness, and stability. In this talk, we present new forms of the Gronwall inequality for normalized fractional integral operators with Mittag-Leffler kernels. Using these newly derived inequalities, we establish existence and uniqueness results for the fractional Cauchy problem involving the Mittag-Leffler kernel. Additionally, we derive comparison principles for related fractional integral inequalities by estimating the normalized derivative of a function at its extreme points. These principles are then applied to obtain pre-norm estimates for the solutions to related linear fractional differential equations. Finally, two examples are provided to demonstrate the effectiveness of the proposed results.

Keywords: Normalized fractional operators; Gronwall inequalities; Maximum principles

2010 Mathematics Subject Classification:26A33; 34A08;39B62; 39B52

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Periodic Sequences generating Equivalent Classes on the unit Circle

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Abstract: In [2], graphs are generated using pairs of parameters (k, n) and (t, m), where t, k, n, and m are natural numbers with n even and m odd. The parameters $a_e = \frac{1}{2}\left(\frac{4k}{n} + 1\right)$ and $a_o = \frac{1}{2}\left(\frac{4t-1}{m} + 1\right)$ are used to generate closed graphs on the unit circle through periodic sequences mapped onto the circle. In this work, we establish a necessary and sufficient condition for determining equivalence classes of parameter pairs that produce identical graphs and corresponding designs. Additionally, special cases of generated graphs are characterized and simulated. Finally, a link to applications of these equivalence classes in Biomathematics and Artificial Intelligence is discussed.

Keywords: Geometric patterns, equivalent classes, periodic sequences.

2010 Mathematics Subject Classification: 34A08; 34B10

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ROLE OF TEMPERATURE-DEPENDENT FEATURES İN WAVE PROPAGATİON THROUGH TRANSVERSELY ISOTROPİC MİCROPOLAR THERMOELASTİC MEDİA

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Abstract:

This research explores the impact of temperature-dependent features on wave propagation in a generalized thermoelastic micropolar transversely isotropic half-space. The study examines how temperature-dependent material properties affect wave characteristics, such as phase velocity and attenuation. Analytical solutions for these variations are derived and graphically represented to highlight their thermal dependence. The analysis is conducted within the framework of the Lord-Shulman (L-S), Green-Lindsay (G-L), and Coupled Thermoelasticity (C-T) theories. Numerical results are presented through graphical illustrations to enhance understanding of the effects of temperature-dependent properties on wave motion.

Keywords: micropolar, temperature dependence, phase velocity, attenuation.

2010 Mathematics Subject Classification:

- **74J05: Linear wave propagation**
- **74A15: Thermodynamics in solid mechanics74A60: Micromechanical theories**
- **74F05: Thermal effects in solid mechanics**
- **35Q74: PDEs in connection with mechanics of deformable solids**

ERROR ANALYSIS OF THE VECTOR PENALTY-PROJECTION METHODS FOR THE TIME-DEPENDENT STOKES EQUATIONS WITH OPEN BOUNDARY CONDITIONS

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Abstract:

We present in this paper a rigorous error analysis of the vector penalty-projection method for solving the time-dependent incompressible Stokes equations with open boundary conditions on part of the boundary. First, we prove the stability of the scheme. Then, we provide an error analysis for the second-order vector penalty-projection method which shows that the convergence rate of the error on the velocity and the pressure is of order 2 in $l^{\infty}(L^2(\Omega))$ and $l^2(L^2(\Omega))$ respectively. In addition, it is shown that the splitting errors of the method varies as O(ε) where ε is a penalty parameter chosen as small as desired. Several numerical tests in agreement with the theoretical results are presented. To the best of our knowledge, this paper provides the first rigorous proof of optimal error estimates for second order splitting schemes with open boundary conditions.

Keywords: Vector penalty-projection methods; open boundary conditions; error estimates.

2010 Mathematics Subject Classification: 65M12, 35Q30, 35Q35, 76D05.

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MATHEMATICAL ANALYSİS OF SURFACE WAVE VİBRATİON İN FGPM COMPOSİTE STRUCTURE

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Abstract:

This paper aims to study the surface wave vibration in a functionally graded piezoelectric plate resting over a homogeneous elastic substrate. The material properties of the piezoelectric plate are assumed as a linear function of depth. Analytical methods of applied mathematics is used to solve the coupled electro-mechanical field equations. Using proper boundary conditions, the dispersion relations are obtained for both electrically open and short circuit cases. By using a particular example, graphical demonstrations are given for phase velocity which expatiate the influence of functional grading and other relevant parameters on the dispersion of wave. The salient outcomes may be utilized as guidelines for the theoretical study of optimization of the performance of surface acoustic wave devices.

Keywords: piezoelectric plate, functionally grading, aysmptotic approximation

2010 Mathematics Subject Classification: 74J05, 74J15, 74J40, 74E30

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Mathematical analysis on vibrations of Piezo-electro-magneto-thermoelastic composite nano-beams under Three Phase Lag model

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Abstract: As the analysis of thermoelastic damping (TED) and frequency shift (FS) in Nano Electro Mechanical Systems (NEMS) stems from the need to improve energy efficiency, enhance sensitivity, achieve frequency stability, and expand the application potential of NEMS devices, the present study comes up with a mathematical framework to investigate TED and FS in nano-scale piezo-electro-magneto thermoelastic (PEMT) composite beams comprising $BaTiO_3$ fibers and $CoFe_2O_4$ matrix. The Green-Naghdi-III Three Phase Lag thermoelastic (TPL) model is applied to examine TED and FS for clamped-free (CF), doubly clamped (CC), clamped simply supported (CS) and, doubly simply supported (SS) beam configurations through the linear Euler-Bernoulli theory. The model assesses the effects of beam dimensions, fiber-to-matrix volume fraction (Ωf), and the first two vibrational modes on TED and FS by computing eigenvalues. A key finding is the identification of critical thickness (CrTh) and critical length (CrLt) where TED and FS behavior undergo significant changes. This study finds a possible potential application in NEMS devices, where small dimensions and high frequencies make TED a dominant damping mechanism.

Keywords: Thermoelastic Damping, Frequency Shift, Piezo-electro-magneto-thermoelastic composite.

2010 Mathematics Subject Classification: 35Q74, 74A05, 74A10, 74B05, 74F05, 74F15, 74E10, 74E15, 74E30, 74H45, 74K10

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MATHEMATICAL AND PHYSICAL ASPECTS FOR THE SOLUTION OF THE INITIAL-VALUE PROBLEM FOR THE SYSTEM DESCRIBING LINEAR THERMODIFFUSION IN THE THERMAL STRESSES THEORY

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Abstract:

The aim of this paper is to present a self-contained introduction to some important aspects, not only mathematical but also physical, of the solution of the initial value problem for linear thermodiffusion in thermal stress theory.

The theory of thermodiffusion is a fusion of the theory of heat conduction and diffusion and the theory of elasticity. It is concerned with the effect of heat and diffusion on the deformation of an elastic solid and the inverse effect of deformation on the thermal and diffusion state of the solid.

Applying the method of Sobolev spaces and based on the constructed matrix of fundamental solutions to this system of equations, we proved the decay rate estimates for the solution of the initial value problem to this system of equations. We also described that the phenomenon of thermodiffusion appears in some materials not only in the civil aviation but also in the air force.

Hydrogen penetrating under the pressure into the steal produce its significant deformation [1]. The thin plate, in which penetrating the hydrogen, flexure itself, the thin plate suffer twist [2]. Also loaded with consequences is influence the heating of the body in the diffusion process [2, 3]. From some experiments we have changed the distribution of humidity in the porous medium under the influence of changing of the temperature field.

In order to accelerate the process of separation of the gas from the metal it is possible to apply the heating of the body, so that the process like carburization or decarburization of the steal comes after the presence of the variable field of the temperature. Such phenomena take place in some materials used in mechanical engineering. Especially in materials used in aeronautics not only in civil aviation but also in the air force [4, 5].

So there are necessary to create the theory including the correlation between the diffusion process - which is described by the chemical potential, the thermal process with is described by the temperature field and the deformation of the body described by the displacement and the state of stress. Such a theory described above is called thermodiffusion in solid body.

Keywords: thermodiffusion, thermal stresses theory, hyperbolic-parabolic partial

2010 Mathematics Subject Classification: 35A07, 35A08, 35L15, 35L45

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PREDICTING THE POSITION OF MATRIX PSEUDOSPECTRUM WITH RESPECT TO A GIVEN COMPLEX DOMAIN

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General area of research: Applied Mathematics

Abstract: Generally speaking, the eigenvalues of matrices in many cases provide excellent insight into the dynamic properties of the observed systems. However, this is not always the case. Given linear dynamical system, if its matrix is normal, the eigenvalues fully explain the dynamic properties (not only the asymptotic ones). However, if this matrix is not normal, the evolution function of the considered dynamical system can have a very different behavior, before the asymptotic is manifested. The feature of the matrix which can explain this transitional behavior is the matrix pseudospectrum. More precisely, ε-pseudospectrum is used to establish spectral properties that are *robust* under matrix perturbations bounded in a given norm by the parameter $\varepsilon > 0$. One such property is certainly the stability of continuous time dynamical systems, see [1]. Since it arises in many practical applications, its robustness is one of the most important questions. The measure of the robustness of stability of a matrix is known as *distance to instability*, cf. [1], and it is usually related to the distance from pseudospectrum to $y-$axis. Since the computation of such value requires solving a nonconvex optimization problem, numerical algorithms need to be employed, see [2, 3, 4, 5, 6], all of them demanding significant computational power, especially in the case of large sparse matrices. Therefore, it is important to find tight (and computationally cheep) pseudospectra localizations, see [7]. Namely, lower bounds for the distance to instability obtained in such way will either be quite precise, or can be used in speeding up the numerical algorithms by providing good starting points. The aim of this presentation is to discuss position of pseudospectra with respect to any given complex domain, not only to open left half-plane. Again, localization of pseudospectra in arbitrary p norm, are in the main focus. Also, due to frequently present block structure of considered matrices, the connection between original block matrix and its comparison matrix (of smaller dimension) will be discussed.

Keywords: pseudospectrum, distance to instability, M-matrices.

2010 Mathematics Subject Classification: 15A18, 15B99, 65F99

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ANALYSIS OF NUMERICAL SOLUTION OF FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract:

Functional differential equations, in particular, delay differential equations are a promising class of differential equations. Delay differential equations ae the one where rate of change of variables depend on both present and past phenomena. Extensive research is being conducted all across the world to explore the advantages of these equations. In this paper, one such aspect of delay differential equations is presented that is analysing the numerical solution and its properties by application of a numerical technique. Further, some interesting results are presented to verify the precision and accuracy of the solution obtained.

Keywords: Numerical method; Functional differential equation

2010 Mathematics Subject Classification: 34K05, 34K06, 65L03

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GS4 – Graph Theory, Coding Theory & Cryptography

COMPARATIVE ANALYSIS OF TOPOLOGY AND ENTROPY CHARACTERIZATION IN AMINAL LINKED COVALENT ORGANIC FRAMEWORKS AND GRAPH ENERGY PREDICTION

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Abstract:

The discovery of novel topologies in covalent organic frameworks (COFs) is a cornerstone of reticular chemistry and materials science, opening new avenues for innovative design. This approach not only provides a systematic framework for constructing covalent-organic architectures but also unveils the complex relationship between structure and function. By introducing new linkage chemistry, we significantly enhance the structural diversity of COFs, imbuing them with captivating properties. In this paper, we examine the graph structural properties of amine-linked COFs, calculate entropy levels using hybrid topological descriptors, and assess their topological complexity. Additionally, these descriptors form the foundation for developing robust statistical regression models, enabling accurate predictions of graph energy in higher-dimensional frameworks. This exploration paves the way for deeper insights and advancements in the design and application of COFs.

Keywords: Covalent organic frameworks; topological descriptors and entropy; graph energy.

2010 Mathematics Subject Classification: 05C07, 05C09, 05C92.

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MATHEMATICAL TECHNIQUES FOR GRAPH DESCRIPTORS, ENTROPIES, SPECTRA, AND PROPERTIES OF OXALATE-BASED METAL ORGANIC FRAMEWORKS

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Abstract:

Metal organic frameworks (MOFs) are indispensable for enhancing the performance of various technologies due to their porous nature and tunable structural architecture. Their versatility enables significant applications in energy storage, catalysis, gas separation, and sensing technologies. Oxalates play a key role in binding metal ions, contributing to the formation of durable and efficient MOFs. This paper investigates the topological spectral properties of oxalate-based MOFs, emphasizing the roles of distance and degree indices in their structural characterization. We have developed innovative computational methods to calculate distance based topological indices for a series of interconnected pentagons, which present unique computational challenges in oxalate-based MOFs. The degree based hybrid indices are involved in calculating the entropies of MOFs, and their structural pattern is discussed using a scaled entropy approach.

Keywords: Distance indices; degree indices; strength-weighted graphs.

2010 Mathematics Subject Classification: 05C09, 05C12, 05C07, 05C92

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MINIMAL WEIGHTED ACYCLIC GRAPHS

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Abstract:

In this talk, we show a completely characterization of all the trees on n vertices with diameter d for which there is a symmetric matrix with nullity $n-d$ and $n-d-1$, respectively. These characterizations cover all recent results proved for the standard $0-1$ adjacency matrices. Here a new technique is developed for the general case, breaking the limitation of star complement technique for the standard $0-1$ adjacency matrices.

Keywords: nullity, tree, diameter

2010 Mathematics Subject Classification: 05C50

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COMPARATIVE STUDIES OF TOPOLOGIAL INDICES OF CHEMICAL STRUCTURES

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Abstract:

Chemical graph theory involves analyzing the topological characteristics of molecules by converting molecular structures into graphs and then computing their molecular properties. A topological index is a non-empirical mathematical value that represents the connectivity between different parts of a chemical structure in graph form. This approach offers a theoretical framework for describing molecular characteristics and predicting properties such as bioactivity, stability, reactivity, and electronic behavior, all of which are tied to the topological properties of molecular frameworks. This study presents a comparative analysis of various chemical structures using different mathematical approaches for wavy zigzag nanoribbons, nanoporous graphenes with [14]annulene pores, and guanidinium and hydrogen carbonate rosette layers. These insights enhance QSAR and QSPR analyses, revealing many observable properties, such as thermal stability, conductivity, and toxicity potential, for these materials.

Keywords: Distance indices; szeged-type indices; strength-weighted graphs.

2010 Mathematics Subject Classification: 05C07, 05C09, 05C12, 05C92

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TOPOLOGICAL INDICES, GRAPH ENTROPIES, AND REGRESSION MODELS OF COVALENT ORGANIC FRAMEWORKS

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Abstract:

Covalent organic frameworks (COFs) represent a class of crystalline porous materials that have captured significant interest across various fields due to their high surface area and tunable pore size. The Kagome lattice COFs are a two-dimensional network of interconnected triangles with hexagonal patterns that provide an excellent platform for designing materials with tailored porosity. We provide insights into their network connectivity through degree-based topological indices that facilitate the predictions of physicochemical properties, biological activities, and network transformations during phase transition phenomena. In this study, we analyze Kagome lattice COFs by computing their topological indices and entropies using bond partitioning techniques and regression models focusing on three types of organic linkers. We also explore the relationships between topological indices and graph entropies through linear and logarithmic regressions, and derive quantitative expressions for scaled entropy to evaluate the order/disorder complexities of these COFs. This approach provides valuable insights into their physicochemical properties and phase transition behaviors.

Keywords: Degree-based topological indices; scaled entropies; regression models.

2010 Mathematics Subject Classification: 05C07, 05C09, 05C92

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ON SZEGED-TYPE INDICES OF ZEOLITE BCT FRAMEWORK

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Abstract:

The Szeged-type indices are graph-theoretical distance-based measure with significant potential applications in QSAR/QSPR (Quantitative Structure-Activity/Property Relationship) studies . These indices are associated with the closeness-based edge function, which are instrumental in quantifying the peripherality of individual bonds. By evaluating the contribution of each bond to the overall structure of the chemical compound, offering a comprehensive characterization of its structural properties. Further, these measures are critical for understanding molecular structure-property relationships, making them valuable in fields like medicinal chemistry, materials science, and bioinformatics. The computation of Szeged-type indices typically employs various techniques, including computer algorithms, numerical interpolation methods, and the classical cut method. Among these, the classical cut method is particularly preferred due to its relatively lower computational complexity and efficiency. However, for certain complex frameworks, such as zeolite BCT (body-centered tetragonal tectosilicate), the cut method faces challenges due to its non-partial cube aspects and the intricate nature of the quotient graph associated with the BCT framework increases the difficulty of accurately computing its Szeged-type indices. To address these challenges, this paper introduces an innovative approach that integrates the cut method with the unique structural attributes of the zeolite BCT framework to compute Szeged-type indices.

Keywords: Szeged-type indices, zeolite BCT, non-partial cube.

2010 Mathematics Subject Classification: 05C92, 92E10.

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LOCALLY MAXIMAL RECOVERABLE CODES

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Abstract: Locally recoverable (LRC) codes came into prominence due to their applications in addressing the problem of data recovery in data storage systems in the event of node failures [1, 2]. They are frequently used in distributed storage systems and cloud storage systems such as Microsoft Azure and Hadoop. Traditionally, the conventional erasure codes, such as Reed-Solomon codes, have been used for data recovery in such systems. However, the recovery process typically requires access to a relatively large number of remaining symbols. On the other hand, LRCs focus on recovering a coordinate value with the help of a small number of other coordinates.

In this talk, we will discuss about a new class of locally recoverable codes (LRCs), called Locally Maximal Recoverable (LMR) codes. Traditional LRCs with (r, δ) locality ensure that each local set can recover up to $\delta - 1$ erasures. However, their recovery capabilities are limited when the number of erasures exceeds this threshold within any local set. LMR codes are a subclass of codes with (r, δ)-locality such that they can correct h additional erasures in any one local set, in addition to having (r, δ)-locality. These codes are capable of recovering all information-theoretically possible erasures in any one of the local sets while keeping the property of (r, δ) locality. Therefore, these codes are better equipped for data recovery than (r, δ) LRCs. We will also discuss a construction of distance optimal cyclic LMR codes. In this construction, we define a code through its parity-check matrix, which is obtained by choosing certain special types of matrices so that the code satisfies the LMR property.

Keywords: LRC, MR codes, LMR codes.

2010 Mathematics Subject Classification: 94B05, 94B15, 94B60.

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Extremal Chemical Trees for Exponential Augmented Zagreb Index

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Abstract:

A significant challenge in combinatorics and graph theory is the characterization of extremal structures concerning graph invariants, particularly within the class of chemical trees. Cruz, Monsalve, and Rada [Applied Mathematics and Computation 380 (2020) 125281] proposed a unified framework for identifying extremal chemical trees for degree-based graph invariants based on graph order. Among these invariants, the exponential augmented Zagreb index (EAZ) is a well-established measure for a graph G, defined as:

$$EAZ(G) = \sum_{v_iv_j \in E(G)} e^{\left(\frac{d_i d_j}{d_i + d_j - 2}\right)^3},$$

where d_i denotes the degree of vertex v_i, and $E(G)$ represents the edge set of G.

However, the unique computational properties of EAZ rendered it unsuitable for inclusion in the previously mentioned unified framework. Consequently, determining the extremal chemical trees for EAZ was identified as an open problem in the same study.

This work addresses the open problem by providing a complete solution. We identify the maximal and minimal chemical trees for the EAZ invariant, categorized by graph order n.

Keywords: Topological index, Exponential augmented Zagreb index, Extremal graph, Chemical tree

AMS subject classification: 05C50, 05C09, 05C92

FACE DEGREE INDICES OF BENZINOID HYDROCARBONS

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Abstract:

Benzenoid hydrocarbons, a notable subclass of polycyclic aromatic hydrocarbons (PAHs), are of significant interest due to their unique properties and applications in fields ranging from pharmaceuticals to environmental science. This study introduces a novel set of topological indices based on the face characteristics of molecular graphs, integrating degree, degree-sum, and hybrid descriptors. These indices were employed to predict key physicochemical properties of 79 benzenoid hydrocarbons, including melting point, flash point, retention index, polarizability, heat capacity, enthalpy of vaporization, molar refraction, and log P. We also demonstrated that these novel face-based indices exhibit strong linear correlations with the studied properties, offering a powerful tool for predicting the physicochemical behavior of benzenoid hydrocarbons. This approach provides a valuable contribution to the development of efficient and accurate predictive models for chemical properties, with implications for drug discovery, material design, and environmental risk assessment.

Keywords: Face degree indices; degree-sum indices; molecular properties.

2010 Mathematics Subject Classification: 05C07, 05C09, 05C92.

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QUANTUM PRIVATE INFORMATION RETRIEVAL FROM REPAIRING REED-SOLOMON CODES

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Abstract: Quantum Private Information Retrieval (QPIR) protocols enable a user to retrieve a classical file from multiple servers by downloading quantum systems without revealing the identity of the file. This ensures both data privacy and security. In [1], a capacity-achieving scheme for QPIR was proposed by combining the star-product scheme by Allaix et al. [4] and the stabilizer QPIR scheme by Song et al. [2]. This combination ensures that the QPIR protocol achieves the maximal achievable ratio of the retrieved file size from the quantum system, thereby optimizing the efficiency and security of the retrieval process. This paper proposes a protocol to optimise the repair bandwidth by using repairing Reed-Solomon codes [5, 3, 6] into QPIR protocols to enable efficient data reconstruction.

Keywords: Private Information Retrieval, Quantum PIR, Repairing Reed-Solomon Codes.

2010 Mathematics Subject Classification: 11T71 Algebraic coding theory, Cryptography

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Weighted topological index of graphs

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Abstract:

The definition of the weighted topological index [1, 2] associated with a degree function ϕ is $\Phi(G) = \sum_{uv \in E(G)} \phi(d_u, d_v)$, where d_u denotes the degree of node u and ϕ satisfies symmetric property $\phi(d_u, d_v) = \phi(d_v, d_u)$. Let R be a commutative ring with unity $1 \neq 0$, the cozero divisor graph associated to R is denoted by $\Gamma'(R)$, and is defined as a simple graph with node set as non-zero non-unit elements of R such that two nodes a and b with $(a \neq b)$ are adjacent iff $a \notin Rb$ and $b \notin Ra$, where aR is the ideal generated by a. Afkhami and Khashyarmanesh [3, 4, 5, 6] in a collection of papers studied the properties of $\Gamma'(R)$ like their graph complements, planarity, identification of commutative rings with forest, star or unicyclic graphs, their relations with comaximal graphs of rings and zero divisor graphs. In this paper, we characterized extremal graphs and presented several results concerning the function $\Phi(G)$ in terms of various graph invariants. Additionally, we characterize the graphs that achieve these bounds and present multiple bounds for $\Phi(G)$ for the class of cozero divisor graphs defined on commutative rings.

Keywords: Topological index, Zero divisor graph, Extremal graph

AMS subject classification: 05C50, 05C09, 05C92, 15A18

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ℓ-intersection Pairs of Constacyclic and Conjucyclic Codes

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Abstract: A pair of linear codes whose intersection is of dimension ℓ, where ℓ is a non-negetive integer, is called an ℓ-intersection pair of codes [1]. This paper focuses on studying ℓ-intersection pairs of λ_i-constacyclic, $i = 1, 2$, and conjucyclic codes. We first characterize an ℓ-intersection pair of λ_i-constacyclic codes. A formula for ℓ has been established in terms of the degrees of the generator polynomials of λ_i-constacyclic codes. This allows obtaining a condition for ℓ-linear complementary pairs (ℓ-LCP) of constacyclic codes. Later, we introduce and characterize ℓ-intersection pairs of conjucyclic codes over $\mathbb{F}_{p^2}$ [2]. The first observation in the process is that there are no non-trivial linear conjucyclic codes over finite fields. So focus on the characterization of additive conjucyclic (ACC) codes. We show that the largest $\mathbb{F}_p$-subcode of an ACC code over $\mathbb{F}_{p^2}$ is cyclic and obtains its generating polynomial. This enables us to find the size of an ACC code. Furthermore, we discuss the trace code of an ACC code and show that it is cyclic. Finally, we determine ℓ-intersection pairs of trace codes of ACC codes over $\mathbb{F}_4$.

Keywords: Linear code, Constacyclic code, Conjucyclic code, ℓ-intersection

2010 Mathematics Subject Classification: 35A01, 65L10, 65L12, 65L20, 65L70

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GS5 - Computational Mathematics & Numerical Analysis

Pressure Poisson splitting scheme for Navier-Stokes Equations with open boundaries

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Abstract: I present a novel time-stepping algorithm for solving incompressible flows in domains with open boundaries. The scheme is based on the pressure Poisson reformulation of the system. A novel boundary condition is proposed for the pressure variable on the open boundary. Unconditional stability is shown for a stabilized scheme. Numerical tests indicate excellent stability and accuracy of the proposed algorithm.

Keywords: Navier-Stokes equations, Pressure Poisson, consistent-splitting.

2010 Mathematics Subject Classification: 65M60, 76D05

Structure preserving reduced-order model for parametric cross-diffusion systems

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Abstract:

In this work, we construct a structure-preserving reduced-order model for the resolution of parametric cross-diffusion systems. Cross-diffusion systems model the evolution of the concentrations or volumic fractions of mixtures composed of different species and often read as nonlinear degenerated parabolic partial differential equations whose numerical resolutions are highly expensive from a computational point of view. We are interested here in cross-diffusion systems which exhibit a so-called entropic structure, in the sense that they can be formally written as gradient flows of a certain entropy functional which is actually a Lyapunov functional of the system. In this work, we propose a new reduced-order modelling method, based on a reduced basis paradigm, in order to accelerate the resolution of parameter-dependent cross-diffusion systems, which preserves, at the level of the reduced-order model, the main mathematical properties of the continuous solution, namely mass conservation, non-negativeness, preservation of the volume-filling property and entropy-entropy dissipation relationship. The theoretical advantages of our approach are confirmed by several numerical experiments.

Keywords: Cross-diffusion systems, finite volumes, Reduced-order-model (ROM)

2010 Mathematics Subject Classification: 65M08, 65M60

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BLOCK GENERALIZED MINIMAL RESIDUAL METHOD

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General area of research: Applied Mathematics, Computational Mathematics

Abstract: We present a new variant of block generalized minimal residual method (GMRES) [3] for linear systems with multiple right-hand sides:

$$AX = B,\ A \in \mathbb{C}^{N\times N},\quad X \in \mathbb{C}^{N\times M}. \tag{1}$$

Previously introduced block versions of the GMRES method [1], [2] can reduce only the initial block in fact. As a result, the need for frequent restarts arises.

The main feature of the presented method is a strategy for block deflation. This allows us to avoid undesirable restarts and keeps all accumulated information instead. As a result, a number of matrix-vector products decreases significantly.

In addition, the presented method can append new right-hand sides to the solving process on the go. This makes it possible to maintain an optimal block size, which balances the overall complexity of the method (and the amount of memory it uses) and the efficiency of operations. Thus the total time for solving the system is reduced.

Keywords: Krylov methods, Generalized minimal residual method, block solvers

2010 Mathematics Subject Classification:

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FUNDAMENTAL SOLUTION-BASED NETWORK FOR SOLVING THE INVERSE CAUCHY PROBLEM OF HOMOGENEOUS DIFFERENTIAL EQUATIONS

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Abstract: The Method of Fundamental Solutions (MFS) has been widely applied to solve boundary value problems. However, the placement of source points remains a significant challenge, limiting its practical applicability, particularly in inverse problem settings. To overcome this limitation, we propose a novel physics-informed RBF neural network: the Fundamental Solution-Based Network. Drawing inspiration from MFS, we use the fundamental solutions of the homogeneous equations as activation functions and place the source points as the centers of the neurons. These source points are trained by minimizing the physics-informed loss function that incorporates the boundary conditions. The governing equations are not embedded in the loss function, avoiding the issue of the disparity in convergence rates between the PDE term and the boundary condition term, which often hampers optimization. Furthermore, the proposed strategy for initial placement of source points is highly adaptable and can be applied to problems with complex boundary geometries. We apply this framework to solve the inverse Cauchy problem for homogeneous differential equations and present numerical experiments to demonstrate the effectiveness and efficiency of the proposed approach.

Keywords: Method of fundamental solution, Physics-informed neural network, inverse Cauchy problem.

2010 Mathematics Subject Classification: 65N21, 65N80

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ON THE IMPROVING OF TT-CROSS APPROXIMATION METHOD

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Abstract:

Tensor train decomposition [1, 2] provides a method to express a tensor with a logarithmic number of parameters. A significant benefit is that most operations involving tensor trains can be reduced to sequences of matrix operations. These characteristics have enabled its effective use in numerous applications.

In practical scenarios, many tensors are impossible to store due to their large number of elements, and they are represented by functions of integer variables instead. This poses a significant challenge: how to obtain a tensor train approximation for such tensors. The TT-cross method [3] is efficient in approximating the tensor using a logarithmic number of evaluations of the functions and ensuring low computational complexity. It selects submatrices of the tensors' associated matrices heuristically and forms an approximation from those selections.

However, the TT-cross approximation presents a few limitations: theoretically, the approximation error of the tensor could potentially be exponentially related to the approximation error of the chosen submatrices; additionally, determining the ranks remains a challenge.

This talk focuses on enhancing the TT-cross approximation by employing error orthogonalization, thereby addressing the challenge of potential exponential error increase, and by introducing a new strategy for rank adaptation.

Keywords: tensor train, approximation

2010 Mathematics Subject Classification: 15A69

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BAYESIAN ANOMALY DETECTION IN VARIABLE-ORDER AND VARIABLE-DIFFUSIVITY FRACTIONAL DIFFUSION MEDIUMS

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Abstract:

Fractional diffusion equations (FDEs) are powerful tools for modeling anomalous diffusion in complex systems, such as fractured media and biological processes, where nonlocal dynamics and spatial heterogeneity are prominent. These equations provide a more accurate representation of such systems compared to classical models but pose significant computational challenges, particularly for spatially varying diffusivity and fractional orders. In this talk I will present a Bayesian inverse problem for FDEs in a 2-dimensional bounded domain with an elliptical anomaly of unknown geometric and physical properties, where the latter are the diffusivity and fractional order fields. To tackle the computational burden of solving dense and ill-conditioned systems, we employ an advanced finite-element scheme incorporating low-rank matrix representations and hierarchical matrices. For parameter estimation, we implement two surrogate-based approaches using polynomial chaos expansions: one constructs a 7-dimensional surrogate for simultaneous inference of geometrical and physical parameters, while the other leverages solution singularities to separately infer geometric features, then constructing a 2-dimensional surrogate to learn the physical parameters and hence reducing the computational cost immensely.

Keywords: Fractional Diffusion Equations, Bayesian Inverse Problem, Surrogate Models.

2010 Mathematics Subject Classification: 35R11, 62F15, 65M60

ADVANCING CONVOLUTION QUADRATURE METHODS FOR NUMMERICAL STABILITY AND THEORETICAL ANALYSIS IN DIFFERENTIAL EQUATIONS

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Abstract:

Convolution quadrature methods have emerged as a critical tool for the numerical approximation of convolution integrals, particularly in the context of solving stiff ordinary differential equations (ODEs), parabolic partial differential equations (PDEs), and hyperbolic initial-boundary value problems. This research aims to explore the dual utility of convolution quadrature methods, focusing on both their robust numerical stability and their theoretical role in analyzing standard time discretization methods such as linear multistep and Runge–Kutta methods.

The study will involve the formulation and analysis of new convolution quadrature rules specifically designed to address the challenges posed by stiff ODEs and PDEs. By deriving discrete analogs of the variation-of-constants formula within the convolution quadrature framework, the research seeks to provide deeper insights into the stability and accuracy of existing time discretization methods.

To validate the developed methods, a comprehensive computational framework will be implemented, enabling extensive numerical experiments across a variety of test cases, including real-world applications in engineering and physics, such as anomalous diffusion processes. The research paves the way for the development of advanced convolution quadrature rules, significant theoretical contributions to the analysis of time discretization methods, and the creation of practical computational tools for solving complex differential equations.

This work promises to contribute significantly to the fields of numerical analysis and applied mathematics by offering both theoretical advancements and practical solutions for the challenges of fractional differential equations.

Keywords: Convolution Quadrature; Time Discretization Methods; Fractional Differential Equations.

2010 Mathematics Subject Classification: 34A45, 34A08, 45D05.

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GS6 – Statistics & Machine Learning

TOWARDS ROBUST PREDICTION USING THE ELLIPTICAL PROCESS FOR REGRESSION

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General area of research: Statistics

Abstract:

In this work, we introduce Elliptical Processes (EPs), a novel Bayesian family of models that extend traditional regression frameworks. EPs generalize Gaussian Processes (GPs) by incorporating non-normal tail distributions, providing greater accuracy in challenging scenarios such as heavy-tailed or outlier-prone data, where GP assumptions break down. We address the complexity of EP inference through the Laplace approximation method, ensuring efficient computation. Preliminary results from synthetic and real-world datasets demonstrate that EPs outperform GPs in capturing non-Gaussian behaviors. Our presentation will offer a comparative analysis of EPs, GPs, and T-Processes, highlighting EPs' superiority in regression problems where GPs are less effective. This work underscores the importance of EPs in real-world applications, particularly when Gaussian assumptions fail.

Keywords: Gaussian process; Elliptical process; Regression.

2010 Mathematics Subject Classification: Primary 62J02; Secondary 60G15.

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GENERATION OF NON-LINEAR SYNTHETIC TIME SERIES OF EEG DATA USING LSTM MODELS

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Abstract:

Electroencephalogram (EEG) signals have been proposed as a valuable source of information for the early detection of Parkinson's disease [1]. EEG signals are examples of time series data, which are recorded and organized in a specific chronological sequence [2]. Processing time series data poses a significant challenge in data science, particularly when the series is long, complex, and nonlinear. Discovering hidden patterns within these series requires advanced methods that combine statistical models with artificial intelligence algorithms [3].

The philosophy of time series forecasting lies in the ability to predict future values based on past observations. Despite the importance of these predictions, their implementation presents many challenges. Time series data often contain noise and missing values, which can compromise data quality and, in turn, reduce prediction accuracy. Additionally, the use of inappropriate models or insufficient data can further diminish the effectiveness of these forecasts [4].

The problem at hand is to classify EEG time series data to determine, through binary classification, whether a person has Parkinson's disease. Classifiers based on artificial intelligence techniques or statistical methods, particularly those based on deep learning, require a large amount of data for effective training and testing. In cases where sufficient data is unavailable, the generation of synthetic data has been proposed. Studies have shown that incorporating synthetic data into the training process can lead to improved performance.

Methods for generating synthetic data can be broadly divided into two main categories: traditional methods and machine learning techniques. Traditional methods include models such as Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA) [5], and Autoregressive Integrated Moving Average (ARIMA) [6]. While effective, these models have imitations. They require smooth and stationary data, which is often not the case in real-world scenarios where time series data tends to be noisy and unstable [7]. With the rapid advancement in AI, researchers have turned to neural networks and deep learning techniques to address the challenges posed by traditional models [8].

In this paper, we use LSTM networks to model the generation of EEG time series, demonstrating their effectiveness in capturing the complex temporal dependencies inherent in neural signals. LSTM networks are particularly well-suited for this task due to their ability to retain long-term patterns and leverage them as memory, making them ideal for managing the dynamic and nonlinear nature of EEG data. As a result, LSTM models can effectively generate synthetic EEG time series.

We have trained and tested LSTM models to generate synthetic EEG time series. The suitability of the approach is demonstrated by the high correlation and the small error between synthetic and original data. Furthermore, the synthetic samples are usuful for training the classifiers, as an example of data augmentation technique.

Keyword: Time series, Synthetic data, LSTM

2010 Mathematics Subject Classification:

62 – Statistics
- **62-10** - Linear Time Series Models (AR, MA, ARMA, ARIMA)
- **62-20** - Time series forecasting and analysis
- **62-40** - Mixed Models for Time Series

68 - Artificial Intelligence and Machine Learning
- **68T** - Artificial Intelligence and Machine Learning
  - **68T05** - Artificial Neural Networks and Deep Learning
  - **68T10** - Regression Models and Predictive Data Analysis
  - **68T20** - Machine Learning for Time Series

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THE IMPACT OF CORPORATE GOVERNANCE ON THE ENVOLMENT IN CORPORATE SOCİAL RESPONSİBİLİTY ACTIVITIES

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Abstract:

Corporate social responsibility (CSR) reflects ethics and corporate actions on three pillars, reflecting on environmental reporting, social features and governance (ESG). CSR is a key factor found in the literature for achieving economic goals and creating wealth.

Companies with higher levels of governance offer greater transparency in the implementation of daily rules and operations. They can make timely adjustments to existing problems in the operation of the company. At the same time, they can avoid petential risks over time, reduce transaction costs and improve the efficiency of production and operation.

Regarding the methodology, the data were collected using the Thomson Reuters database, and their processing was done using the Stata 15 statistical threshold. The study was conducted over a ten year period (2013-2023) on the states of the European Union.

The results of our research capture a strong bond between high corporate governance and engagement in CSR activities.Our research is of interest to companies that currently do not have the obligation to report ESG activities. The study can also impact stakeholders (banking companies, investors, clients, the state).

The participation of companies in social responsibility activities is beneficial for establishing trust between the company and stakeholders and can also strengthen the sense of responsibility of employees towards companies. The established relationship is a strong point of the companies and also helps them to understand their shortcomings in the strategic objectives to be developed.

Keywords: governance, CSR, ESG.

2010 Mathematics Subject Classification:
- o 62-07 Data analysis;
- o 62F03 Hypothesis testing;
- o 62J05 Linear regression.

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UNBIASED PARAMETER ESTIMATION FOR PARTIALLY OBSERVED DIFFUSION

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Abstract:

We consider the problem of estimating static parameters for a partially observed diffusion process X_t with discrete-time noisy observations $\{Y_t\}_{t=1}^T$ over a fixed time interval $[0, T]$. The diffusion satisfies the equation

$$dX_t = a_\theta(X_t)dt + \sigma(X_t)dW_t$$

where W_t is a standard Brownian motion, and the goal is to estimate θ. In particular, we assume that one must time-discretize the partially observed diffusion process and work with the model with bias and consider maximizing the resulting log-likelihood of the discrete model. Using a novel double randomization scheme, based upon Markovian stochastic approximation we develop a new method to unbiasedly estimate the static parameters, that is, to obtain the maximum likelihood estimator for the continuous model with no time discretization bias. Under assumptions we prove that our estimator is unbiased and investigate the method in several numerical examples, showing that it can empirically out-perform existing unbiased methodology.

Keywords: Unbiased estimation, Markovian stochastic approximation, Diffusion processes.

2010 Mathematics Subject Classification: 60J22, 62M05, 65C40, 62M20.

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OPTIMAL CONTROL OF THE FALSE DISCOVERY EXCEEDANCE IN LARGE-SCALE DIRECTIONAL MULTIPLE TESTING

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Abstract: In large-scale multiple hypothesis testing, controlling the false discovery exceedance (FDX) is emerging as a compelling alternative to the widely employed false discovery rate (FDR) when the false discovery proportion (FDP) exhibits high variability. Existing methods for FDX control have mainly focused on non-directional discovery within a two-group model framework. However, this approach can significantly compromise the reliability of FDX control when directional decisions are required, especially in cases when the signal-to-noise ratio is low. Furthermore, the theoretical optimality of FDX control has not been thoroughly explored. In this paper, we introduce an empirical Bayes approach tailored for directional FDX control within a three-group model. We demonstrate that an oracle decision rule, which ranks and thresholds a directional version of the local false discovery rate (lfdr), achieves optimality by maximizing power while adhering to the directional FDX constraint. We also propose a data-driven procedure to mimic the oracle rule in practical applications and establish its asymptotic optimality. Through extensive simulation studies and a real-data application, we show the superior performance of our method in directional FDX control.

Keywords: Empirical Bayes; False discovery exceedance; Multiple hypothesis testing.

2010 Mathematics Subject Classification: 62J15

A Two-Stage Approach for Bayesian Network Structure Discovery under Data Uncertainty

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Abstract:

Bayesian networks are probabilistic graph models that represent the conditional dependencies between random variables through a directed acyclic graph (DAG). Most Bayesian network structure learning algorithms make the idealistic assumption that the observed data is error-free, and measurement errors present in real data can degrade the accuracy of the trained network. To account for the presence of irregular measurement errors in the data, this study proposes a two-stage approach to reduce the negative impact of these errors on network learning.

The two-stage approach proposed in this study is to first learn the initial structure of the network with variables that are assumed to be error-free, and then extend the structure by adding variables that contain measurement errors. Simulation studies show that the proposed method outperforms existing general approaches, especially when noisy variables are in extreme positions. Application results using real-world thermal environment data demonstrate that the two-stage approach can more clearly identify the relationships between variables, contributing to a more plausible explanation of the counterintuitive correlations found in previous studies.

Keywords: Bayesian Network, Structure Learning, Thermal Environment

2010 Mathematics Subject Classification: 62-09

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LOMAX-PEARSON TYPE VII DISTRIBUTION WITH APPLICATIONS TO FINANCIAL STOCK RETURNS

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Abstract:

The class of elliptically contoured distributions (ECD) includes distributions with heavier tails than the normal distribution. In this research, a new subclass of ECD has been introduced. It is constructed as a scale mixture of Lomax distribution with Pearson type VII distribution. The mathematical properties of the Lomax-Pearson VII (LP7) distribution, such as stochastic representation, characteristic function, have been investigated. In addition, the usefulness of LP7 has been illustrated through an empirical study of financial stock returns. Under Akaike information criterion, the study revealed that LP7 distribution outperformed the normal distribution.

Keywords: Elliptically contoured distributions, scale mixture, Lomax-Pearson 7.

2010 Mathematics Subject Classification: 62E15, 62P05

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THE İMPACT OF AUDİT AND REPORTİNG STANDARDS ON THE DYNAMİC MODEL OF FOREİGN PORTFOLİO İNVESTMENT

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Abstract:

This study examines the dynamics of international portfolio investments, focusing on the influence of financial reporting quality on foreign investment flows. It explores how countries with higher Strength of Audit and Reporting Standards (SARS) attract more foreign portfolio investments due to increased transparency and reliability in financial statements. Leveraging a dynamic panel regression model with fixed time effects, the study analyzes data from 84 countries between 2007 and 2017. The Generalized Method of Moments (GMM) addresses endogeneity, ensuring robust estimation of parameters. Results indicate that higher SARS correlates with increased foreign investment flows, particularly in developed economies. The findings emphasize the importance of strong institutional frameworks, infrastructure, and international financial reporting standards in promoting capital flows. Robustness checks confirm the stability of the regression results, supporting the conclusion that SARS plays a critical role in shaping investment patterns and enhancing global economic stability. These insights are valuable for policymakers, regulators, and investors aiming to improve investment climates.

Keywords: International Portfolio Investments (FPI), Strength of Audit and Reporting Standards (SARS), Generalized Method of Moments (GMM)

2010 Mathematics Subject Classification:

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ESTIMATION FOR CONDITIONAL MOMENT MODELS BASED ON MARTINGALE DIFFERENCE DIVERGENCE

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Abstract:

The abstract is not to exceed one page. We provide a new estimation method for conditional moment models via the martingale difference divergence (MDD). Our MDD-based estimation method is formed in the framework of a continuum of unconditional moment restrictions. Unlike the existing estimation methods in this framework, the MDD-based estimation method adopts a non-integrable weighting function, which could capture more information from unconditional moment restrictions than the integrable weighting function to enhance the estimation efficiency. Due to the nature of shift-invariance in MDD, our MDD-based estimation method can not identify the intercept parameters. To overcome this identification issue, we further provide a two-step estimation procedure for the model with intercept parameters. Under regularity conditions, we establish the asymptotics of the proposed estimators, which are not only easy-to-implement with expectation-based asymptotic variances, but also applicable to time series data with an unspecified form of conditional heteroskedasticity. Finally, we illustrate the usefulness of the proposed estimators by simulations and two real examples.

Keywords: Conditional moment models; Martingale difference divergence; Time series model estimation

2010 Mathematics Subject Classification: 62H12

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Discovering causal relationships among financial variables associated with firm value using a dynamic Bayesian network

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This study investigated the causal relationships among financial variables associated with firm value using a causal dynamic Bayesian network (CDBN), an extension of the basic Bayesian network that captures both temporal and contemporaneous causal relationships. The CDBN model was constructed using a panel dataset of listed manufacturing companies in Korea over a 14-year period (2009–2022). By visualizing the interactions between financial factors, the model makes it easy to understand their dynamic and instantaneous relationships, offering valuable insights into corporate finance. Key findings in the model include evidence of autocorrelation in all dynamic variables, a lagged feedback loop between the intangible assets ratio and firm value, the widespread impact of the COVID-19 pandemic on the financial sector, and important causal relationships involving key financial metrics such as the fixed asset ratio, firm value, and return on assets ratio.

BERRY-ESSEEN BOUNDS FOR DEGENERATE U-STATISTICS WITH APPLICATION TO THE DISTANCE CORRELATION

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Abstract: Let $X_1, X_2, ..., X_n$ be independent and identically distributed random vectors, $T_n = T_n(X_1, X_2, ..., X_n)$ be a degenerate U-statistic, and $\Delta_n = \Delta_n(X_1, X_2, ..., X_n)$ be a remainder term. In this paper, we establish a Berry-Esseen type theorem for $T_n + \Delta_n$ by an exchangeable pair approach. As an application, a sharp error bound of normal distribution approximation for the distance correlation is obtained, which improves some results in Gao, Fan, Lv and Shao (2021). This talk is based on a joint work with Songhao Liu and Hao Shi.

Keywords: Berry-Esseen bound, degenerate U-statistics, distance correlation

2010 Mathematics Subject Classification: 60F05, 62E17

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EXPLORING A NEW THEORETICAL SCHEME TO CONSTRUCT SIMPLE ASYMMETRIC BIVARIATE COPULAS: APPLICATION TO ABU DHABI CLIMATOLOGICAL DATA

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Abstract:

Bivariate copulas play a central role in modeling the dependence structure between two random variables and serve as a fundamental tool in various applied fields. In this article, we develop a new theoretical framework aimed at constructing simple asymmetric bivariate copulas of the form $C(u,v) = uv\,[\phi(v) + u(1-\phi(v))]$, $(u,v) \in [0,1]^2$. This framework relies on a tuning univariate function to achieve the desired asymmetry. We study this pioneering scheme, emphasizing its theoretical foundations and illustrating it with several examples. More precisely, we establish important properties of the proposed copulas and derive analytical expressions for concordance measures such as Spearman's rho, Kendall's tau, Gini's gamma and Blomqvist's beta. In addition, we investigate the estimation procedure for the dependence parameter using the maximum likelihood approach. Finally, we perform a simulation study to evaluate the performance of the proposed estimator. A real climatological dataset from the city of Abu Dhabi is used to demonstrate the applicability of the proposed copulas, with very convincing results.

Keywords: Asymmetric copulas; dependence models; concordance measures.

2010 Mathematics Subject Classification: 60E05, 62E15, 62H99

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LLM-ENHANCED DEEP REINFORCEMENT LEARNING FOR AUTOMATED TRADING

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Abstract:

Deep reinforcement learning (DRL) has emerged as a powerful framework for decision-making, enabling agents to optimize objectives through trial-and-error interactions with their environment. However, its application to high-dimensional, complex domains such as stock market analysis presents substantial challenges, including the need for sophisticated architectures to process multimodal data and significant computational resources. This work explores the integration of large language models (LLMs) into the DRL paradigm, leveraging their capacity to capture intricate patterns, utilize extensive pre-trained knowledge, and generalize across diverse tasks. We propose a novel architecture that combines LLMs and DRL, where LLMs extract actionable insights from multimodal financial data—comprising historical prices, technical indicators, and sentiment analysis—while a deep Q-network (DQN) optimizes decision-making to maximize cumulative rewards. Preliminary experiments demonstrate the effectiveness of LLMs in stock market prediction, showcasing their ability to accurately interpret patterns in historical data and public sentiment derived from news sources. We hypothesize that fine-tuning pre-trained LLMs on domain-specific financial datasets and embedding them into DRL frameworks can significantly enhance learning efficiency and predictive accuracy, reducing the reliance on additional neural architectures for complex data processing. Furthermore, our approach aims to outperform existing benchmarks while mitigating computational costs and energy consumption associated with training traditional deep learning-based trading systems. These advancements have the potential to drive the development of scalable, efficient, and sustainable solutions in financial technology, contributing to more accessible and resource-conscious automated trading systems.

Keywords: Deep reinforcement learning (DRL), Large language models (LLMs), Deep Q-network (DQN)

2010 Mathematics Subject Classification: Artificial intelligence (68T99)

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IMPROVED CUSUM CONTROL CHART FOR PROCESS VARIANCE USING BAYESIAN METHODS

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Abstract: Monitoring process variability is a critical objective in statistical process control. Bayesian control charts (BCC) have emerged as effective statistical tools for examining manufacturing processes and managing process variability. This technique is particularly suited for handling parameter uncertainty in the manufacturing domain. This article introduces a novel Bayesian control chart (CC) scheme, utilizing a three-parameter logarithmic transformation combined with a cumulative sum (CUSUM) approach to monitor process sample variance. Using posterior estimates, Bayesian CUSUM charts are constructed for process variance with different conjugate priors, specifically inverted Chi-square and inverted Gamma distributions. The newly developed Bayesian charts outperform traditional control charts in tracking process variance. A simulation study confirms that the proposed Bayesian CUSUM CCs are more effective than conventional classical CUSUM charts for variance monitoring, excelling in detecting shifts in the variance of a normally distributed process. Additionally, real manufacturing data is analyzed to validate the simulation findings, further supporting the proposed method's effectiveness.

Keywords: CUSUM Chart; Posterior Distribution; Aerospace Manufacturing.

2010 Mathematics Subject Classification: Statistics

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THE BACKGROUND STRESS ESTIMATED FROM STRESS ROTATION USING A BAYESIAN APPROACH

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Abstract:

Iwata (2018)[1] developed a Bayesian approach with a Markov chain Monte Carlo method to estimating a spatial stress pattern from *P*-wave first motions. Then, Iwata (2023) [2] modified this approach to enhance the spatial resolution of the estimation; the Delaunay triangulation was incorporated to represent the spatial stress pattern. The modified approach was also applied to a dataset taken from the aftershocks of the 2000 Tottori Western earthquake in Japan and clarified the stress orientation (stress rotation) difference across the main fault on the north and south edges of the area where a large slip occurred.

This study estimated the background stress in the aftershock area of the 2000 Western Tottori earthquake using the stress rotation as described above. For simplicity, it was assumed that the orientations of the maximum and minimum principal stress axes of the background stress are EW and NS, respectively. Therefore, the orientation of the intermediate principal stress axis is assumed to be vertical. The spatial stress distribution caused by the mainshock (coseismic stress changes) was calculated from the slip model estimated by the waveform inversion by Iwata et al. (2000) [3] and the program of Okada (1992) [4]. Then, a particular value of the differential stress was given, and the spatial pattern of the principal stress axes was computed as the sum of the background stress and the coseismic stress changes. The value of the differential stress was optimized to fit the computed spatial pattern with the above Bayesian method; the difference between the two stress patterns was measured with the Kagan angle proposed in Kagan (1991) [5], and the total difference was minimized. The minimization (optimization) was conducted through the maximum likelihood method, where the Kagan angle is assumed to follow the von Mises distribution. As a result, the optimal differential stress was estimated at around 45 MPa.

Keywords: stress rotation, directional statistics, Bayesian method

2010 Mathematics Subject Classification: 62-07 Data analysis

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Bayesian Approach for Hidden Markov Regression Model

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Abstract:

The Hidden Markov Model (HMM) is a statistical model based on a Markov chain, where observed values are determined by unobservable hidden states that follow a Markov process. Due to these characteristics, HMMs are widely used for analyzing data with patterns that change over time.

By modeling the observation process associated with each state using a regression model, HMMs can effectively describe and interpret data, even when it exhibits complex observational mechanisms. The intricate structural relationships between states, observation processes, and the resulting observed values can be represented using a hierarchical model. This enables a Bayesian approach by assuming distributions for each parameter.

In this study, data with observation processes varying over time were generated to evaluate whether the proposed model effectively captures such dynamics through a simulation study. Additionally, an analysis of data from Seoul was conducted to examine the differential effects of meteorological factors on fine particulate matter (PM2.5).

Keywords: Bayesisan Hierarchical Modeling, Hidden Markov Model, Lasso Regression

2010 Mathematics Subject Classification: 62M05

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Identification and Estimation of Change Points in Factor Models for High-Dimensional Time Series Data

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Abstract:

The focus is on estimating and identifying a factor model for high-dimensional time series that contains structural breaks in the factor loading space at unknown time points. We first study the case when there is one change point in factor loadings, and propose a consistent estimator for the structural break location. We show that the proposed estimators for change-point location and loading spaces are consistent when the number of factors is correctly estimated or overestimated. The algorithm for multiple change-point detection is also developed. A distinguishing feature of the proposed method is that it is specifically designed for the changes in the factor loading space and the stationarity assumption is not imposed on either the factor or noise process, while most existing methods for change-point detection of high-dimensional time series with/without a factor structure require the data to be stationary or close to a stationary process between two change points, which is rather restrictive. Numerical experiments, including a Monte Carlo simulation and a real data application, are presented to illustrate the proposed estimators perform well.

Keywords: Factor models; high-dimensional time series; change points.

2010 Mathematics Subject Classification: 62 Statistics

POISSON-CHARLIER POLYNOMIAL DISTRIBUTION AND ITS APPLICATIONS

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Abstract: We introduce a class of Poisson-Charlier Polynomial (PCP) distribution models to study count data. These models permit a wide range of count data characteristics, including multimodal densities, overdispersion, underdispersion, skewness, and varying tail behaviors. We thoroughly examine the statistical properties of the PCP distribution, such as the characteristic function, and develop regression models to analyze complex datasets. Comprehensive tests, including score tests and likelihood-ratio tests, are conducted to validate the robustness and applicability of these models. The practical utility of our approach is demonstrated through the analysis of hotel booking cancellation datasets, highlighting the models' flexibility and effectiveness in capturing intricate data features.

Keywords: Generalize Poisson Distribution, Poisson-Charlier Polynomial, Regression Models

2010 Mathematics Subject Classification: 62E15

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BEYOND LİNEAR REGRESSİON: ENHANCİNG PREDİCTİVE ACCURACY İN STOCK PRİCE PREDİCTİON USİNG ENSEMBLE METHODS

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Abstract:

This study compares the predictive performance of ensemble methods, specifically, decision trees, random forests, and gradient boosting, focusing on their tuning and performance in predicting stock prices using a 500-company dataset from Yahoo Finance. The analysis focuses on understanding stock price fluctuations, trading volume variability, and the distribution of dividends and stock splits. The ensemble methods were evaluated using root mean square error, mean absolute percentage error, and the coefficient of determination to comprehensively assess prediction accuracy and error reduction. The results revealed that decision trees outperformed random forests and gradient boosting in terms of accuracy and error rate after hyperparameter tuning. When tuned, random forests demonstrated high performance, effectively capturing intricate and nonlinear interactions within the dataset. However, gradient boosting exhibited mixed results. This study provides insights into the strengths of each model and highlights the advantages of ensemble methods in enhancing stock price prediction accuracy. The findings provide a foundation for further enhancements in financial forecasting models.

Keywords: Ensemble Methods, Machine learning, Financial Forecasting Models.

2010 Mathematics Subject Classification: 62J07, 62H30, 62P20, 62M45, 68T05, 91G10.

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