Larger than One

Submission deadline: August 28^{th} 2024

Let a, b, c, d be positive numbers less than one. How many of the following numbers can be larger than one?

$4a(1-b), 4b(1-c), 4c(1-d), 4d(1-a)$

The problem was solved by

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Discussion:

Suppose that each term is larger than one. Then, it follows that

$$4a(1-b)\cdot 4b(1-c)\cdot 4c(1-d)\cdot 4d(1-a)>1.$$

By rearranging terms we get that

$$4a(1-a)\cdot 4b(1-b)\cdot 4c(1-c)\cdot 4d(1-d)>1.$$

Consider the function $f(x)=4x(1-x)$. It is clear that $f(x)\leq 1$, for all x. Thus, it is impossible for $4a(1-a)\cdot 4b(1-b)\cdot 4c(1-c)\cdot 4d(1-d)$ to be greater than 1. Therefore, our initial assumption is wrong. Hence all of the given numbers cannot be larger than 1.

If $a=\frac{1}{2}, b=\frac{3}{8}, c=\frac{3}{10}, d=\frac{1}{12}$, then 3 of the given numbers are larger than 1.

Thus, at most 3 of the given numbers can be larger than 1.