Add and Subtract

Submission deadline: October 28th 2024

If

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{90}$$

find

$$1 - \frac{1}{2^4} + \frac{1}{3^4} - \frac{1}{4^4} + \frac{1}{5^4} - \frac{1}{6^4} + \cdots$$

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Discussion:
We start with

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{90} \tag{1}$$

Multiplying both sides of (1) by $\frac{1}{2^4}$ yields that

$$\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \frac{1}{8^4} + \frac{1}{10^4} + \cdots = \frac{1}{2^4} \frac{\pi^4}{90} \tag{2}$$

Now subtracting two times (2) from (1) results in

$$1 - \frac{1}{2^4} + \frac{1}{3^4} - \frac{1}{4^4} + \frac{1}{5^4} - \frac{1}{6^4} + \cdots = \frac{7}{8} \cdot \frac{\pi^4}{90}$$