Cosines

Submission deadline: August 28^{th} 2024

Find

$$\cos(1) + \cos(2) + \cdots + \cos(2024)$$

The problem was solved by

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Discussion:

Since $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, we have that $\sum_{n=1}^{2024} \cos(n)$ is the real part of $\sum_{n=1}^{2024} e^{in}$, which clearly is a geometric series with ratio e^i. Therefore,

$$\sum_{n=1}^{2024} e^{in} = e^i \frac{1 - e^{2024i}}{1 - e^i}$$

Computing the real part of $e^i \frac{1 - e^{2024i}}{1 - e^i}$, yields that

$$\cos(1) + \cos(2) + \cdots + \cos(2024) = \frac{\cos(1) + \cos(2024) - \cos(2025) - 1}{2(1 - \cos(1))}$$